MAKTAB O'QUVCHILARI UCHUN OLIMPIADA MASALALARINI YECHISHDA KOSHI TENGSIZLIGINING QO'LLANILISHI.

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Koshi tengsizligini isbotlash usuli

Yevklid tekisligida $\vec{a}$ va $\vec{b}$ vektorlar mos holda $\vec{a}(a_1;a_2)$ hamda $\vec{b}(b_1;b_2)$ ko'rinishdagi koordinatalari bilan berilgan bo'lsin.

1.1-ta'rif. Yevklid tekisligida $\vec{a}$ va $\vec{b}$ vektorlarning skalyar ko'paytmasi deb $\vec{a}\cdot\vec{b}=a_1 a_2+b_1 b_2$ (1.1)

ko'rinishidagi songa aytiladi. Shuningdek, bu skalyar ko'paytmaning vektorlarning uzunliklari hamda ular orasidagi burchaklari kosinuslari orqali ifodasi ham mavjud bo'lib $\vec{a}\cdot\vec{b}=|\vec{a}|\cdot|\vec{b}|\cdot\cos\alpha$ (1.2) ga teng. Ularni o'zaro tenglashtirish mumkin

$$\vec{a}\cdot\vec{b}=a_1 a_2+b_1 b_2=|\vec{a}|\cdot|\vec{b}|\cdot\cos\alpha \quad (1.3)$$

Skalyar ko'paytma modulini baholaylik. Agar oxirgi ifodadan $\cos\alpha$ ni olib tashlasak, $\vec{a}\cdot\vec{b}=a_1 a_2+b_1 b_2=|\vec{a}|\cdot|\vec{b}|\cdot\cos\alpha\le|\vec{a}|\cdot|\vec{b}|$ (1.4) kelib chiqadi. Bu erda $\vec{a}\cdot\vec{b}=a_1 a_2+b_1 b_2=|\vec{a}|\cdot|\vec{b}|\cdot\cos\alpha\le|\vec{a}|\cdot|\vec{b}|$ sharti o'rinli o'rinli bo'lishligi uchun $\cos\alpha=1$ shart bajarilishi kerak. Qolgan holatlarda $|\vec{a}|\cdot|\vec{b}|$ son birdan kichik songa ko'paytirilganligi uchun

$|\vec{a}|\cdot|\vec{b}|\cdot\cos\alpha\le|\vec{a}|\cdot|\vec{b}|$ (1.5) tengsizlik o'rinli bo'ladi. Endi (3.1.4) ifodadagi $\vec{a}\cdot\vec{b}$ ko'paytma o'rniga va vektorlarning $|\vec{a}|\cdot|\vec{b}|$ uzunliklari orqali o'rinlariga ularning koordinatalar

ifodalarini qo'yamiz. U holda quyidagi tengsizliklar hosil bo'ladi:

$$\vec{a}\cdot\vec{b}=a_1 a_2+b_1 b_2=|\vec{a}|\cdot|\vec{b}|\le\sqrt{a_1^2+a_2^2}\cdot\sqrt{b_1^2+b_2^2}$$

bundan esa $(a_1 a_2+b_1 b_2)^2\le(a_1^2+a_2^2)(b_1^2+b_2^2)$ (1.6)

tengsizlik kelib chiqadi. (1.1.6) tengsizlik to‘rtta a_1,a_2,b_1,b_2 sonlar uchun

Koshi tengsizligining eng sodda yoki xususiy ko‘rinishi deb ataladi.

Agar (3.1.6) tengsizlikda $a_1 b_2-b_1 a_2=0$ shart bajarilsa

$$(a_1 b_1+a_2 b_2)^2\le(a_1^2+a_2^2)(b_1^2+b_2^2)$$

kelib chiqadi. Endi Koshi tengsizligining to‘la ko‘rinishi

$$(a_1 b_1+a_2 b_2+a_3 b_3+\cdots+a_n b_n)^2$$

$$\le(a_1^2+a_2^2+a_3^2+\cdots+a_n^2)(b_1^2+b_2^2+b_3^2+\cdots+b_n^2) \quad (1.6)$$

ni hosil qilamiz.

1-usul Faraz qilaylik $a_1,a_2,a_3,\ldots a_n,b_1,b_{(2,)} b_3,\ldots b_n\ge0$ bo’lsin . Ma’lumki, tengsiz-liklarni isbotlashda belgilashlar kiritish usulidan foydalanish mumkin.

Quyidagi belgilashlarni kiritamiz.

$$x_k=\sqrt{(a_1^2+a_2^2+a_3^2+\cdots+a_n^2)(b_1^2+b_2^2+b_3^2+\cdots+b_n^2)},$$

$$x_{(k+1)}=\sqrt{(a_1^2+a_2^2+a_3^2+\cdots+a_n^2+a_{(n+1)}^2)(b_1^2+b_2^2+b_3^2+\cdots+b_n^2+b_{(n+1)}^2)}$$

Ikkinchi ifodalarda qavslarni ochib soddalashtirsak:

$$x_{(k+1)}=\sqrt{(a_1^2+a_2^2+a_3^2+\cdots+a_n^2+a_{(n+1)}^2)(b_1^2+b_2^2+b_3^2+\cdots+b_n^2+b_{(n+1)}^2)}$$

$$=\sqrt{\left(\left(\sqrt{a_1^2+a_2^2+a_3^2+\cdots+a_n^2}\right)^2+a_{(k+1)}^2\right)\left(\left(\sqrt{b_1^2+b_2^2+b_3^2+\cdots+b_n^2}\right)^2+b_{(k+1)}^2\right)}\ge$$

$$\sqrt{\left(\left(\sqrt{(a_1^2+a_2^2+a_3^2+\cdots+a_n^2)(b_1^2+b_2^2+b_3^2+\cdots+b_n^2)}\right)^2+a_{k+1}\cdot b_{k+1}\right)^2}$$

$$=x_k+a_{k+1}\cdot b_{k+1} \quad (1.6)$$

Xulosa qilib aytganda $x_{k+1}\geq x_k+a_{k+1}\cdot b_{k+1}$ tengsizlik hosil bo'ladi. Agar eski belgilashlarga qaytsak,

$$\sqrt{(a_1^2+a_2^2+a_3^2+\cdots+a_n^2)(b_1^2+b_2^2+b_3^2+\cdots+b_n^2)} \geq a_1 a_2$$

Tengsizlikning har ikkala tomonini kvadratga ko'tarsak .

$$(a_1^2+a_2^2+a_3^2+\cdots+a_n^2)(b_1^2+b_2^2+b_3^2+\cdots+b_n^2)$$

$$\geq(a_1 b_1+a_2 b_2+a_3 b_3+\cdots+a_n b_n)^2 \quad (1.7)$$ Hosil bo'ladi.

Tengsizlik $a_1,a_2,a_3,\ldots a_n,b_1,b_{2,}\ b_3,\ldots\ b_n\geq 0$ shartlar bajarilganda Koshi tengsizligining to'la ko'rinishidan iborat . Agar $a_1,a_2,a_3,\ldots a_n,b_1,b_{2,}\ b_3,\ldots\ b_n$ ixtiyoriy sonlar bo'lsa , u holda (3.1.7) tengsizlikning quyidagi ko'rinishida yozish mumkin:

$$(a_1^2+a_2^2+a_3^2+\cdots+a_n^2)(b_1^2+b_2^2+b_3^2+\cdots+b_n^2)$$

$$= (|a_1|^2+|a_2|^2+|a_3|^2+\cdots+|a_n|^2)(|b_1|^2+|b_2|^2+|b_3|^2+\cdots+|b_n|^2)$$

$$\geq (|a_1 b_1|+|a_2 b_2|+|a_3 b_3|+\cdots+|a_n n|)^2$$

$$\geq|a_1 b_1+a_2 b_2+a_3 b_3+\cdots+a_n b_n|^2= (a_1 b_1+a_2 b_2+a_3 b_3+\cdots+a_n b_n)^2$$

1.1 - misol : Agarda $a_1,a_2,a_{3,}\ldots a_n\in[a;b]$ $0<a<b$ bo'lsa,

$$n^2\le(a_1+a_2+a_3+\dots+a_n)\left(\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}+\dots+\frac{1}{a_n}\right)\le\frac{(a+b)^2}{4ab}n^2$$

isbotlang.

a) $(a_1+a_2+a_3+\dots+a_n)\left(\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}+\dots+\frac{1}{a_n}\right)\ge n\sqrt[n]{a_1\cdot a_2\cdot a_3\cdot\ldots\cdot a_n}\cdot n\sqrt[n]{\frac{1}{a_1}\cdot\frac{1}{a_2}\cdot\frac{1}{a_3}\cdot\ldots\cdot\frac{1}{a_n}}=n^2$ bilan $n^2\le(a_1+a_2+a_3+\dots+a_n)\left(\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}+\dots+\frac{1}{a_n}\right)$

ekanligi isbotlandi

b) $(a_1-a)(a_1-b)\le0,\quad a_1-a\le0, a_1-b\ge0$

$a_1^2+ab\le a_1(a+b)\rightarrow a_1+\frac{ab}{a_1}\le a+b$

$$+\begin{cases}a_1+\frac{ab}{a_1}\le a+b\\ a_2+\frac{ab}{a_2}\le a+b\ldots\\ a_n+\frac{ab}{a_n}\le a+b\end{cases}$$

$(a_1+a_2+a_3+\dots+a_n)+ab\left(\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}+\dots+\frac{1}{a_n}\right)\le n(a+b)$

Koshi tengsizligi bo'yicha $x+y\ge2\sqrt{xy}$

$$\underbrace{(a_1+a_2+a_3+\dots+a_n)}_{x}+\underbrace{ab\left(\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}+\dots+\frac{1}{a_n}\right)}_{y}\ge 2\sqrt{(a_1+a_2+a_3+\dots+a_n)\cdot ab\left(\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}+\dots+\frac{1}{a_n}\right)}$$

bundan

$$2\sqrt{(a_1+a_2+a_3+\dots+a_n)\cdot ab\left(\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}+\dots+\frac{1}{a_n}\right)}\le$$

$$(a_1+a_2+a_3+\dots+a_n)+ab\left(\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}+\dots+\frac{1}{a_n}\right)\le n(a+b)$$

ekanligi keltirildi.

$$2\sqrt{(a_1+a_2+a_3+\cdots+a_n)\cdot ab\left(\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}+\cdots+\frac{1}{a_n}\right)}\le n(a+b)$$

$$4(a_1+a_2+a_3+\cdots+a_n)\cdot ab\left(\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}+\cdots+\frac{1}{a_n}\right)\le (a+b)^2 n^2$$

$$(a_1+a_2+a_3+\cdots+a_n)\cdot\left(\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}+\cdots+\frac{1}{a_n}\right)\le \frac{(a+b)^2}{4ab} n^2$$

tengsizlik isbotlandi.

Adabiyotlar

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