VOLTERRA INTEGRAL TENGLAMALARINI TAQRIBIY YECHISH.

Aniq fanlarni absissial va ordinatal aloqadorlikda o'quvchi kreativ faoliyatini rivojlantirish.

Мухторов Беҳруз Қудратулла ўғли

Термез давлат муҳандислик ва агротехнологиялар университети академик лицейи математика фани ўқитувчиси

Integral tenglamalar nazariyasidan ma'lumki, agar yadro uzluksiz bo'lsa, ikkinchi tur Volterra integral tenglamasi

$$y(x) - \lambda \int_a^{\infty} K(x,s)\, y(s) ds = f(x)$$

yagona uzluksiz yechimga ega. Bu yechimni quyidagi shakldа qidiramiz

$$y(x) = \sum_{k=0}^{\infty} \lambda^k \varphi_k(x)$$

Ushbu qatorni tenglamaga qo'yib, X ning bir xil darajalari oldidagi koeffitsiyent--larni tenglashtirib quyidagilarni olamiz:

$\varphi_0(x) = f(x);$ $\varphi_{k+1}(x) = \int_a^{\infty} K(x,s)\varphi_k(s)ds.$

Agar $N = \max_{a \le xb} |f(x)|,$ $M = \max_R |K(x,s)|,$

bo'lsa, u holda $|\varphi_k(x)| \le \frac{M^k (b-a)^k N}{k!}$.

Bundan, tenglamaning taqribiy yechimi sifatida qatorning qismiy yig'indisini olsak:

$$Y_n(x) = \sum_{i=0}^{n} \lambda^k \varphi_k(x)$$

u holda uning xatosini quyidagicha baholash mumkin:

$$|y(x)-Y_n(x)|=\left|\sum_{i=0}^{n}\lambda^k\varphi_k(x)\right|\leq\sum_{k-n+1}^{\infty}\frac{|\lambda|^kM^k(b-a)^kN}{k!}$$

Qo'polroq, lekin ayni paytda oddiyroq xato bahosi quyidagi bilan belgilanadi, biz quyidagilarni olamiz:

$$|y(x)-Y_n(x)|\leq\frac{L^{n+1}N}{(n+1)!}\left\{1+\frac{L}{n+2}+\frac{L^3}{(n+2)(n+3)}+\cdots\right\}$$

Katta qavsdagi qatorni

$$1+\frac{L}{n+2}+\left(\frac{L}{n+2}\right)^2+\left(\frac{L}{n+2}\right)^3+\cdots$$

bilan almashtirib quyidagi bahoni olamiz:

$$|y(x)-Y_n(x)|\leq\frac{L^{n+1}N}{(n+1)!}\frac{1}{1-\frac{L}{n+2}}$$

keyin biz quyidagi taxminni olamiz:

$$\varphi_{n+1,k}=\int_a^{x_k}K(x_k,s)\varphi_n(s)ds$$
$$\approx\frac{h}{2}\left[K_{k_0}\varphi_{n_0}+2\left(K_{k_1}\varphi_{n_1}+K_{k_2}\varphi_{n_2}+\cdots+K_{k,k-1}\varphi_{n,k-1}\right)\right.$$
$$\left.+K_{kk}\varphi_{nk}\right],$$

yoki

$$\bar{\varphi}_{n+1,k}=\frac{h}{2}\left[K_{k_0}\bar{\varphi}_{n_0}+2\left(K_{k_1}\bar{\varphi}_{n_1}+K_{k_2}\bar{\varphi}_{n_2}+\cdots+K_{k,k-1}\bar{\varphi}_{n,k-1}\right)+\right.$$
$$\left.K_{kk}\bar{\varphi}_{nk}\right]\quad(k=0,1,2,\dots,s)$$

Hisoblab chiqqach, biz integral tenglamanung yechimining taqribiy qiymatlarini tugun nuqtalarda quyidagi formulalar bo'yicha topamiz:

$$Y_{n,k}=\sum_{i=0}^{n}\lambda^i\bar{\varphi}_{i,k}\quad(k=0,1,2,\dots,s)$$

Umumlashtirilgan Simpson formulasidan foydalanganda segmentni nuqtalar bo'yicha teng qismlarga ajratamiz. Keyin, integralni hisoblash uchun Simpson formulasini qo'llaymiz

$$\varphi_{n+1,2k} = \int_a^{x_{2k}} K(x_{2k}, s)\varphi_n(s)ds.$$

Quyidagilarga egamiz

$$\bar{\varphi}_{n+1,2k} = \frac{h}{3}\{K_{2k_0}\bar{\varphi}_{n_0} + 4(K_{2k_1}\bar{\varphi}_{n_1} + K_{2k_3}\bar{\varphi}_{n_3} + \cdots + K_{2k,2k-1}\bar{\varphi}_{n,2k-1}) + 2(K_{2k_2}\bar{\varphi}_{n_2} + K_{2k_4}\bar{\varphi}_{n_4} + \cdots + K_{2k,2k-2}\bar{\varphi}_{n,2k-2}) + K_{2k,2k}\bar{\varphi}_{n,2k}\} \quad (n = 0, 1, 2, \dots; \ k = 1, 2, \dots, s)$$

Toq bo'lganlar uchun k ning qiymatlarini interpolyatsiya qilish orqali topish kerak bo'ladi. Tenglamanung taqribiy yechimi uchun tenglamadagi integralni qandaydir kvadratura formulasi bo'yicha chekli yig'indiga bevosita almashtirish usulidan ham foydalanish mumkin. Masalan, umumlashtirilgan formuladan foydalanganda

trapezoid, segmentni nuqtalar bo'yicha qismlarga bo'lib, biz quyidagilarga ega bo'lamiz:

$$y_k - \lambda \int_a^{x_k} \approx y_k - \frac{h\lambda}{2} f(x_k)\, K(x_k, s)y(s)ds \approx$$

$$\approx [K_{k_0}y_0 + 2(K_{k_1}y_1 + K_{k_2}y_2 + \cdots + K_{k,k-1}y_{k-1}) + K_{k,k}y_k]$$

yoki

$$y_k - \frac{h\lambda}{2}[K_{k_0}y_0 + 2(K_{k_1}y_1 + K_{k_2}y_2 + \cdots + K_{k,k-1}y_{k-1}) + K_{k,k}y_k] - f(x_k) = 0$$

bundan

$$Y_k = \frac{1}{1 - \frac{\lambda h}{2}K_{kk}}\left\{f_k + \frac{\lambda h}{2}K_{k_0}y_0 + h\lambda \sum_{i=1}^{k-1} K_{k_i}y_i\right\}$$

Shunday qilib, bosqichma-bosqich biz barcha qiymatlarni topamiz birinchi turdagi Volterra integral tenglamalariga kelsak

$$\lambda \int_a^x K(x, s)y(s)ds = f(x)$$

keyin yadro uzluksiz differensiallanadi degan qo'shimcha faraz qilibuning funksiyalarini ikkinchi turdagi Volterra integral tenglamasiga keltirish mumkin. Haqiqatan ham, tenglamani differensiallashda biz quyidagilarga ega bo'lamiz:

$$\lambda K(x,y)y(x) + \lambda \int_a^x K'_x(x,s)y(s)ds = f'(x)$$

va ikkinchi turdagi Volterra integral tenglamasining yechimi bo'ladi

$$y(x) + \int_a^x \frac{K'_x(x,s)}{K(x,x)} y(s)ds = \frac{1}{\lambda}\frac{f'(x)}{K(x,x)}$$

Misol. Integral tenglamaning yechimini toping

$$y(x) - \int_0^x e^{-x-s} y(s)ds = \frac{e^{-x} + e^{-3x}}{2}$$

Echish: Birinchi yo'l. Yechimni

$$y(x) \approx Y_4(x) = \varphi_0(x) + \varphi_1(x) + \varphi_2(x) + \varphi_3(x) + \varphi_4(x)$$

bunda

$$\varphi_0(x) = f(x); \qquad \varphi_k(x) = \int_0^\infty K(x,s)\varphi_{k-1}(s)ds$$

ko’rinishda izlaymiz. Integrallash natijasida biz quyidagilarni olamiz:

$$Y_4(x) = \frac{1}{3840}[3839e^{-x} + 5e^{-3x} - 10e^{-5x} - 10e^{-7x} - 5e^{-9x} + e^{-11x}]$$

Bu tenglamaning aniq yechimi

$$y(x) = e^{-x}$$

Taqqoslash uchun biz aniq echimning qiymatlarini va taxminiy echimini taqdim etamiz

$$y(0) = 1{,}00000, \qquad y(1) = 0{,}36788$$

$$Y_4(0) = 1{,}00000, \qquad Y_4(1) = 0{,}36783$$

ЛИТЕРАТУРА

1. Бахвалов Н.С. Численные методи. М.: “Наука”, 1975. 622 с.

2. Михлин С.Г.,Смолицский Х.Л. Приближенные методы решения дифференциальных и интегралных уравнений. М.: Изд-во “Наука”, 1965. 384 с.

3. Крылов В.И. Приближенное вычисление интегралов . М.: Изд-во “Наука”, 1967. 500 с.