SOLVING A SYSTEM OF EQUATIONS USING THE POLAR COORDINATE SYSTEM

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Annotation: The article provides comprehensive information about the polar coordinate system and the generalized polar coordinate system. It presents a general understanding of the fundamental relationships between the polar and Cartesian coordinates of a point, along with several worked examples. The study explores how to solve various equations and systems of equations using the polar coordinate system. Additionally, a methodological recommendation is offered on using the polar coordinate system to enhance the effectiveness of teaching.

Key words: equation, domain of the equation, Cartesian coordinate system, polar coordinate system, generalized polar coordinate system, polar radius of a point, polar coordinates of a point, law of cosines, distance between points, graph of the distance between points.

General Information about the Polar Coordinate System

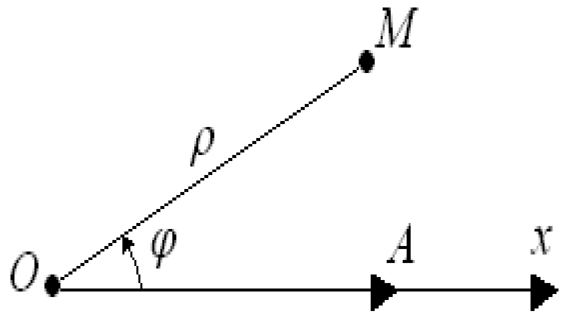
As we know, to determine the position of a point, the rectangular Cartesian coordinate system is commonly used. However, in addition to this system, there are several other coordinate systems as well.

One of them is the polar coordinate system, which offers significant convenience in solving many problems and exercises. In this system, to determine the position of a point on a plane, it is defined by a pole (a reference point), a ray starting from this point called the polar axis, and a scale for measuring distances.

In addition, the definition of the polar system must specify which direction of rotation around the point is considered positive. Usually, counterclockwise rotations (opposite the direction of a clock's hands) are considered positive.

If the polar angle moves in the direction opposite to the clock's hands from the polar axis, it is considered positive; conversely, if it moves in the direction of the clock's hands, it is considered negative. We agree to interpret the angle φ as a trigonometric angle, meaning that we consider it with its sign and additive precision $\pm 2\pi k, k \in Z$. The pair $M(\rho; \varphi)$ is called the polar coordinates of a point M, and it is written in diagrams accordingly. The value of the polar angle M that satisfies a certain inequality $-\pi < \varphi < \pi$ is called the principal value of the polar angle.

According to the above, for polar coordinates, the inequalities $\rho \geq 0$ and $-\pi < \varphi < \pi$ usually apply. However, this condition is not mandatory. In general, ρ and φ can take values across the full range, i.e., $-\infty;+\infty$. When the angle φ is allowed to vary from $-\infty$ to $+\infty$, the coordinate system is referred to as the generalized polar coordinate system.



1-graph.

Graph of the Polar Coordinate System

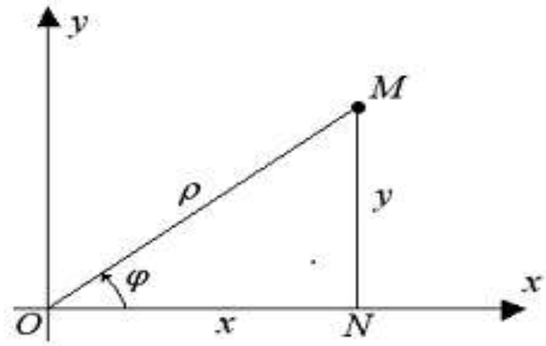
In the polar coordinate system, the position of any point M on the plane is determined by the distance between that point and the pole O, and by the angle between the ray OM and the polar axis. The distance $OM = \rho$ is called the polar radius of the point M, and the angle $\angle AOM = \varphi$ is called the polar angle of the point.

The angle is considered with its sign, and is measured with additive precision $\pm 2\pi k$ (i.e., as in trigonometry). Thus, the polar coordinates of the point M are written as $M(\rho;\varphi)$.

From all possible values that the angle M can take, a specific value that satisfies a certain inequality $-\pi < \varphi \le \pi$ is selected, and this is called the principal value of the polar angle. If the polar coordinates ρ and φ are allowed to vary in the range $-\infty$ to $+\infty$, then the system is called the generalized polar coordinate system.

Graph of the Relationship Between the Polar and Cartesian Coordinate Systems

Now let us consider the following problem: Given the polar coordinates of a point, find its Cartesian coordinates; and conversely, given the Cartesian coordinates, determine the corresponding polar coordinates.



2-graph

Suppose the pole of the polar coordinate system coincides with the origin of the rectangular Cartesian coordinate system, and the polar axis coincides with the positive half of the x-axis (see graph 2). Let M be an arbitrary point on the plane, with $(x;y)$ as its Cartesian coordinates and $(\rho;\varphi)$ as its polar coordinates. The following relationship exists between these coordinates:

$$\begin{cases} x = \rho\cos\varphi \\ y = \rho\sin\varphi \end{cases} \quad (1)$$

Based on these formulas, we can also write the inverse relationships as:

$$x^2 + y^2 = \rho^2 \quad (2)$$

$$\varphi = arctg\frac{y}{x} \quad (3)$$

These equations express the conversion between polar and Cartesian coordinates.

Problem. Find the distance between the points $M_1(\rho_1;\varphi_1)$ and $M_2(\rho_2;\varphi_2)$ in the polar coordinate system.

Solution: According to the law of cosines, the distance d between two points in polar coordinates is given by:

$d^2 = OM_1^2 + OM_2^2 - 2OM_1 \cdot OM_2 \cdot \cos\varphi$ here $\varphi = \varphi_2 - \varphi_1$

$d^2 = \rho_1^2 + \rho_2^2 - 2\rho_1 \cdot \rho_2 \cdot \cos(\varphi_2 - \varphi_1)$ or

$d = \sqrt{\rho_1^2 + \rho_2^2 - 2\rho_1 \cdot \rho_2 \cdot \cos(\varphi_2 - \varphi_1)}$

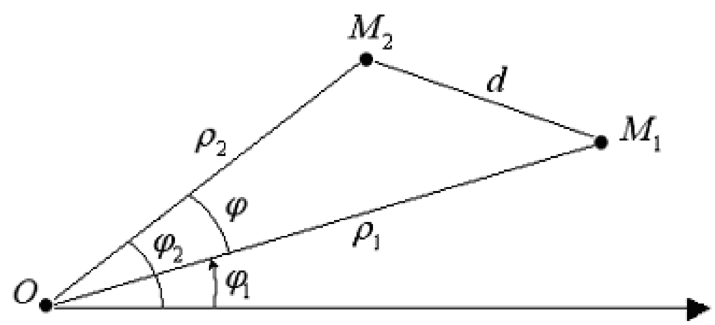
(3-graph).

Graph of the Distance Between Points M_1 and M_2 in the Polar Coordinate System

Solving Equations Using the Polar Coordinate System

1 - example: Solve the following equation:

$$\sqrt[3]{3x+5} - \sqrt{x+3} = 0$$



3-graph.

Solution: To solve this equation using the polar coordinate system, we convert the Cartesian coordinates (x,y)(x, y)(x,y) into polar coordinates using the relations: $\sqrt[3]{3x+5} = r\sin\beta$, $\sqrt{x+3} = r\cos\beta$

$$\begin{cases} 3x+5 = r^3\sin^3\beta \\ 3x+9 = 3r^2\cos^2\beta \end{cases}$$

$$\begin{cases} r\sin\beta - r\cos\beta = 0 \\ r^3\sin^3\beta - 3r^2\cos^2\beta = -4 \end{cases}$$

$r\sin\beta = r\cos\beta$

$\beta=\frac{\pi}{4}+2\pi k$

$r\sin\beta=t$

$3r^2cos^2\beta-r^3sin^3\beta=4$

$-t^3+3t^2-4=0$

$t^3-3t^2+4=0$

$t^3+t^2-4t^2+4=0$

$t^2(t+1)-4(t^2-1)=0$

$(t+1)(t^2-4t+4)=0$

$r\sin\beta=2$; $r\sin\frac{\pi}{4}=2$ $r=\frac{2}{\frac{\sqrt{2}}{2}}=\frac{4}{\sqrt{2}}$

$t+1=0$, $t_1=-1$

$t^2-4t+4=(t-2)^2$

$X+3\left(\frac{4}{\sqrt{2}}\right)^2*\left(\frac{\sqrt{2}}{2}\right)^2$; $x+3=\frac{16}{2}*\frac{2}{4}=4$

$X=4-3=1$

Answer: x=1

2- example: Solve the following equation:

$$\sqrt{\mathbf{1}+\sqrt{\boldsymbol{x}}}+\sqrt{\mathbf{1}-\sqrt{\boldsymbol{x}}}=\mathbf{2}$$

Solution: Domain of the equation:

$\begin{cases} 1+\sqrt{x} \geq 0 \\ 1-\sqrt{x} \geq 0 \end{cases}; x \geq 0; x \leq 1$

To solve the equation, we introduce a new substitution (change of variables).

$\begin{cases} \sqrt{1+\sqrt{x}} = r\, sina \\ \sqrt{1-\sqrt{x}} = r\, cosa \end{cases} \Leftrightarrow \begin{cases} 1+\sqrt{x} = r^2 sin^2 a \\ 1-\sqrt{x} = r^2 cos^2 a \end{cases}$

$2 = r^2 sin^2 a + r^2 cos^2 a$

$\begin{cases} r\, sina + r\, cosa = 2 \\ r^2 sin^2 a + r^2 cos^2 a = 2 \end{cases} \Leftrightarrow \begin{cases} r(\, sina + \, cosa) = 2 \\ r^2 (sin^2 a + cos^2 a) = 2 \end{cases} \Leftrightarrow$

$\begin{cases} r(\, sina + \, cosa) = 2 \\ r^2 = 2 \end{cases}$

From $sin^2 a + cos^2 a = 1$ we get

$Cosa + sina = \sqrt{2} \sin\left(\frac{\pi}{4} + a\right)$

$= r\sqrt{2} \sin\left(\frac{\pi}{4} + a\right) = 2$

$r^2 = 2$

$r_1 = \sqrt{2}$; $r_2 = -\sqrt{2}$

$\sqrt{2}\sqrt{2} \sin\left(\frac{\pi}{4} + a\right) = 2 \Leftrightarrow \sin\left(\frac{\pi}{4} + a\right) = 1 \Leftrightarrow \frac{\pi}{4} + a = \frac{\pi}{2} + 2k\pi \Leftrightarrow$

$a = \frac{\pi}{4} + 2k\pi$.

$\sqrt{1+\sqrt{x}} = \sqrt{2} sin\frac{\pi}{4},\ 1+\sqrt{x} = 2sin^2\frac{\pi}{4}$

$\sqrt{x} = 2\left(\frac{\sqrt{2}}{2}\right)^2 - 1;\ x = 0$

Answer: x=0

Solving a System of Equations Using the Polar Coordinate System

5-example: Solve the system of equations:

$$\begin{cases} \boldsymbol{(u^2+v^2)(u+v)=15uv} \\ \boldsymbol{(u^4+v^4)(u^2+v^2)=85u^2v^2} \end{cases}$$

Solution: $\begin{cases} (u^2+v^2)^2(u^2+v^2+2uv)=225(uv)^2 \\ ((u^2+v^2)^2-2u^2v^2)(u^2+v^2)=85(uv)^2 \end{cases}$

$$\begin{cases} u=r\cos\varphi \\ v=r\sin\varphi \end{cases}$$

We denote the variables accordingly and transform the system into the polar coordinate system, then substitute based on the given definitions.

$$\begin{cases} r^4(r^2+2r^2\cos\varphi\sin\varphi)=225r^4sin^2\varphi cos^2\varphi \\ r^6(1-2sin^2\varphi cos^2\varphi)=85r^4sin^2\varphi cos^2\varphi \end{cases} \Longrightarrow$$

$$\begin{cases} r^2(1+sin2\varphi)=\frac{225}{4}sin^2 2\varphi \\ r^2\left(1-\frac{1}{2}sin^2 2\varphi\right)=\frac{85}{4}sin^2 2\varphi \end{cases}$$

We divide the two equations in the system **term by term**.
Using trigonometric identities, we obtain the expressions necessary for determining the unknown coefficients.

$$\frac{1+\sin2\varphi}{1-\frac{1}{2}sin^2 2\varphi}=\frac{45}{17} \Longrightarrow 17+17sin2\varphi=45-\frac{45}{2}sin^2 2\varphi$$

$$45sin^2 2\varphi+34sin2\varphi-56=0$$

$$sin2\varphi=t$$

$$45t^2+34t-56=0$$

$$t_1=-\frac{14}{9}; \quad t_2=\frac{4}{5}$$

$sin2\varphi \neq -\frac{14}{9}$; $sin2\varphi = \frac{4}{5}$; $\cos 2\varphi = \sqrt{1 - sin^2 2\varphi} = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$;

$\cos 2\varphi = 1 - 2\sin^2\varphi = \frac{3}{5} \Longrightarrow \sin\varphi = \frac{1}{\sqrt{5}}$; $\cos\varphi = \frac{2}{\sqrt{5}}$

Putting angles into system, we find *r*.

$r^2(1 + sin2\varphi) = \frac{225}{4} sin^2 2\varphi$

$r^2\left(1 + \frac{4}{5}\right) = \frac{225}{4} \cdot \frac{16}{25} \Longrightarrow \ r = 2\sqrt{5}$ putting into denotions, we find ***u*** and ***v***.

$u = r\cos\varphi = 2\sqrt{5} \cdot \frac{2}{\sqrt{5}} = 4$
$v = r\sin\varphi = 2\sqrt{5} \cdot \frac{1}{\sqrt{5}} = 2$

Since the variables are mutually symmetric, the point (2,4)(2, 4)(2,4) is also a solution of the system. Moreover, since the given system is both multiplicative and symmetric, the point (0,0)(0, 0)(0,0) also satisfies the system and is therefore a solution.

Answer: (0;0), (4;2), (2;4)

6-example: Solve:

$$\begin{cases} \sqrt[3]{\boldsymbol{x - y}} = \sqrt{\boldsymbol{x - y}} \\ \sqrt[3]{\boldsymbol{x + y}} = \sqrt{\boldsymbol{x + y - 4}} \end{cases}$$

Solution:

$\begin{cases} \sqrt{x - y} = r\cos\varphi \\ \sqrt{x + y - 4} = r\sin\varphi \end{cases}$ We denote accordingly and convert the system into the polar coordinate system, then substitute the expressions based on these definitions.

$\Longrightarrow \begin{cases} x - y = r^2 cos^2\varphi \\ x + y = 4 + r^2 sin^2\varphi \end{cases}$

$$\begin{cases} \sqrt[3]{r^2cos^2\varphi} = r\cos\varphi \\ \sqrt[3]{r^2sin^2\varphi + 4} = r\sin\varphi \end{cases} \Rightarrow \begin{cases} r^2cos^2\varphi = r^3cos^3\varphi \\ r^2sin^2\varphi + 4 = r^3sin^3\varphi \end{cases} \Rightarrow$$

$$\begin{cases} r^2cos^2\varphi(1 - r\cos\varphi) = 0 \\ r^3sin^3\varphi - r^2sin^2\varphi - 4 = 0 \end{cases} \Rightarrow \begin{cases} r_1cos\varphi_1 = 0 \\ r_2cos\varphi_2 = 1 \end{cases}$$

$$r^3sin^3\varphi - r^2sin^2\varphi - 4 = 0$$

$$r\sin\varphi = t$$

$$t^3 - t^2 - 4 = 0$$

$$t^3 - 8 - t^2 + 4 = 0$$

(t-2)($t^2 + 2t + 4$)-(t-2)(t+2)=0

(t-2)($t^2 + t + 2$)=0;   t=2; $\Rightarrow$ $r\sin\varphi = 2$

$$\begin{cases} r_1cos\varphi_1 = 0 \\ r_1sin\varphi_1 = 2 \end{cases} \text{ va } \begin{cases} r_2cos\varphi_2 = 1 \\ r_2sin\varphi_2 = 2 \end{cases}$$

We determine the unknown coefficients.

I. $\begin{cases} \sqrt{x-y} = 0 \\ \sqrt{x+y-4} = 2 \end{cases} \Rightarrow + \begin{cases} x - y = 0 \\ x + y = 8 \end{cases}$ 2x=8; $\begin{cases} x_1 = 4 \\ y_1 = 4 \end{cases}$

II. $\begin{cases} \sqrt{x-y} = 1 \\ \sqrt{x+y-4} = 2 \end{cases} \Rightarrow + \begin{cases} x - y = 1 \\ x + y = 8 \end{cases} \Rightarrow \begin{cases} x_2 = 4{,}5 \\ y_2 = 3{,}5 \end{cases}$

Answer: (4; 4); (4,5; 3,5)

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