APPLICATIONS OF TRIGONOMETRIC FUNCTIONS TO PROBLEM SOLVING

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Annotation: The article gives extensive attention to the applications of trigonometric functions to solving geometric problems, in which various types of geometric problems are solved in several ways using the definitions and properties of trigonometric functions, and geometric problems related to proof are applied to trigonometry and proven in several different ways . The examples and problems presented in the article clearly demonstrate the application of trigonometry to solving various problems in practical concrete examples and problems .

Key words: Trigonometric equation, trigonometric inequality, trigonometric function, algebraic equation , quadratic equation, quadratic root, projection , hypotenuse of a right triangle, altitude of a right triangle, median of a right triangle, bisector of a right triangle.

Trigonometric equations and methods for solving them

Basic methods for solving equations. Trigonometric equations in terms of trigonometric functions of the unknown argument

R(z)= a_0 z^n+a_1 z^(n-1)+⋯+a_(n-1) 〖z+a〗_n=0 (1)

can be reduced to an algebraic equation of the form, where z represents one of the functions sin **λ** x, cos **λ** x, tg **λ** x, ctg **λ** x. In solving trigonometric equations such as algebraic equation (1), methods such as introducing a new unknown, factoring, etc. are used. The process proceeds to solving one of the simplest trigonometric equations. The following cases mainly occur when solving trigonometric equations:

R(f (x)) = 0, the expression f (x) is written under the trigonometric function symbol R, which depends on x. By substituting f (x) = z, the equation can be reduced to one of the simplest trigonometric equations R (z) = 0. Its roots z = are put z_ione by one into f (x) = z , and the values of x are summed.

Example 1. sin⁡〖(10x+π/8)=√(3)/2〗 Solve the equation.

Solution. In our example, f(x)=10x + π/8. The equation

Substituting 10x + π/8= z yields the equation sin z =√3/2

Its solution is: z= (-1)^k π/3+kπ,kϵZ. This is put into 10x + π/8= z and the answer is found: x=1/10 (-π/8+(-1)^k π/3+kπ),kϵZ.

Example 2. We solve the equation tg (x^2+6x+π/6) = √3/3.

Solution.

z=x^2+6x++π/6

substitution kiritamlz. The equation becomes tgz = .√3/3

From him

$z=\pi/6++k\pi$

we find.

In that case

$x^2+6x+\pi/6 =\pi/4+k\pi,k\in Z.$

no

$x^2+6x-k\pi= =o,k\in Z.$

Roots of a quadratic equation

$x=-3\pm \sqrt{(9+k\pi)},k\in Z$,where $9 + k \boldsymbol{\pi} \geq 0$ or $k \geq -9/\pi=-2,86$,

i.e. k= - 2; -1; 0; 1; ⋯ .

Answer: $x= - 3\pm\sqrt{(9+k\pi)},k\in Z,k\geq -2.$

sin⁡〖x=sin⁡〖α,〗 〗 cos⁡〖x=cos⁡α 〗 and tgx=tg **α** equations. These equations can be solved using the formulas x= respectively 〖(-1)〗^k α+kπ,k∈Z,x==±α+2nπ,n∈Z,x=α+mπ,m∈Z.

Example 3. cos⁡〖(5x-45^0)=cos⁡〖(2x+60^0)〗 〗 Solve the equation.

Solution. $45^0=\pm(2x+60^0)+360^0 k,k\in Z$We solve the 5x- equations

$5x-45^0=+(2x+60^0)+360^0 k,k\in Z$

from equality,

$x=35^0+120^0 k,k\in Z$

solution group,

$5x-45^0=-(2x+60^0)+360^0 k, k\in Z$

from equality,

$x=1/7\,(-15^0+360^0 k), k\in Z$

we find the group of solutions.

And so,

$x=35^0+120^0 k, k\in Z; x=1/7\,(-15^0+360^0 k), k\in Z.$

Example 4. $(\sin x)^2=\sin(6x-5)$ we solve the equation.

Solution.

$x^2=(-1)^k(6x-5)+k\pi, k\in Z$

The equation is formed. If k is even, i.e. $k=2n$, $n\in Z$

$x^2=6x-5+2n\pi, n\in Z$

The quadratic equation is derived. Its solution is

$x_{1,2}=3\pm\sqrt{9-(5-2n\pi)}, n\in Z, n\geq[-2/\pi].$

If k is odd, i.e. $k=2m+1$, in $m\in Z$

$x^2=-6x+5+(2m+1)\pi, \quad m\in Z$

will be visible and from this

$x_{1,2}=-3\pm\sqrt{9+(5+2(m+1)\pi)}, m\in Z, m\geq[-(14+\pi)/2\pi].$

$f(R(x))=0$, the trigonometric function R is under the symbol of another function f . Substituting $R(x)=z$ reduces the problem to solving the equation $f(z)=0$. We form a set of equations for the roots of this equation $z_1, z_{(2,\cdots)}$. Solving it solves the problem.

Equations and inequalities involving inverse trigonometric functions.

In solving equations involving inverse trigonometric functions, the fact that the values of trigonometric functions with the same name for equal arguments are equal is used, that is, the property of trigonometric functions being single-valued. In many cases, it is possible to form a simpler equation (for example, an algebraic equation) than the given equation by equating the values of trigonometric functions with the same name for equal arguments given in the form of arcfunctions. The resulting equation is not necessarily equal to the given equation, since the equality of the values of trigonometric functions with the same name does not imply the equality of the arguments of this function.

Example 5. arcsin〖3/5 x+arc sin〖4/5=arc sin〖x 〗 〗 〗 We solve the equation.

Solution. The domain of the equation is x.|3/5 x|≤1,|4/5 x|≤1,|x|≤1

The set of values for which the inequalities are simultaneously satisfied consists of {x: | x | < 1}.

We equate the sines of the left and right sides of the given equation 〖 sin〗 〖(arc sin〖3/5 x+arc sin〖4/5 x〗 〗)=sin〖(arc sinx).〗 〗

Using 3/5 x·√(1-16/25 x^2)+4/5 x·√(1-9/25 x^2)=x.the formula for the sine of the sum and the identities sin〖(arc sinα)=α,〗 cos〖(arc sinα)=√(1-α^2)〗 (here), |α|≤1we get the equation. This equation x_1=0,x_1,2=±1has roots . Substituting each of them directly into the equation, we see that these roots satisfy the equation. For example, for x=1

arcsin〖3/5+arc sin〖4/5=arc sin〖3/5+arc cos〖3/5=π/2.〗 〗 〗 〗

Example 6. 〖(arc sin x)〗^3+〖(arc cos x)〗^3=π^(3)We solve the equation.

Solution.

a^3+b^3=〖(a+b)〗^3-3ab(a+b)

Using the identity and arcsin 〖x+arc cos 〖x=π/2〗 〗 assuming that , we get the following equation:

(π/2)^3-3·π/2·arc sin 〖x·arc cos 〖x=π^3 〗 〗

yoki arcsin 〖x·arc cos 〖x=-7/12 π^2.〗 〗

〖y=arccos〗 〖x,〗 (|y|≤π/2)if we say,y^2-π/2 y-(7π^2)/12=0

A quadratic equation is formed. π/2Since the roots of this quadratic equation are greater than in absolute value, the given equation has no solution.

Example 7. 〖2arc 〖(sin〗 〖x)〗 〗^2-5arc sin 〖x+2=0 〗 We solve the equation.

Solution . arcsin 〖x=z〗 substitution 〖2z〗^2-5z+2=0makes the given equation a quadratic equation. Its roots z_1=0,5, 〖 z〗_2=2. But z 2 roots

- π/2≤arc sin 〖x≤(π)/(2)〗 does not satisfy the condition of being. z_(1)by root

arcsin 〖x=1/(2)〗 we construct the equation. The solution to this equation is 〖x=sin〗 〖0,5.〗

Now let's look at examples of inequalities involving arcfunctions.

Example 〖arctg〗^2 x-4arctgx+3>0 8. We solve the inequality.

Solution. If we assume arctanx = y , the given inequality is as follows

takes the form: $y^2-4y+3>0$.This inequality is satisfied when $y < 1$ or $y > 3$. Returning to the old variable, we have the inequalities arctanx < 1 and arctanx > 3. The inequality arctanx< 1 has solutions xϵ(-∞;tan1), and the inequality arctanx > 3 has solutions xϵ(tan3;+∞).

Answer: (-∞; tan1) ∪ (tan3 ;+∞):

Applications of trigonometric functions to solving geometric problems .

The hypotenuse of a right-angled triangle is b and one acute angle is α. Find the projections of the altitude dropped from the right angle onto the legs.

AC=b,∠BAC=α,BE=FD=u,BF=ED=vIf we say, it is AD=xtaken as. DC=a-xThen ΔDCFfrom:DF/DC=cosα

or $u/(a-x)=\cos\alpha$ (1)

From ΔBDF: FD/BF=tgαoru/v=tgα, (2)

From ΔAED: ED/AD=sinαorv/x=sin α. (3)

(1),(2)and (3)solved together, $x=a\cos\alpha,$

$v=x\sin\alpha$ or $v=a\sin\alpha(\cos\alpha)^2,$

$u=v\,tg\,\alpha$ or $u=v(\sin\alpha)^2\cos\alpha.$

Therefore, BA $a\sin\alpha(\cos\alpha)^2$is equal to the projection, and BE a $(\sin\alpha)^2\cos\alpha$is equal to the projection.

In a right-angled triangle, h is the altitude to the hypotenuse and m is the median. Determine the smallest angle of this triangle.

From ΔBCE:

〖m^2=a^2+〖(c/2)〗^2-2a(c/2)cos〗x (2)

(where BE= c/2);

2) From ΔBCD: h/a=sin〖x, a=h/sinx ;〗 (3)

3) From ΔABC: a/c=cos〖x.〗 Putting (2) into this:

c=h/sin〖x cosx 〗 . (4)

We put (3) and (4) into (2):

m^2=h^2/〖(sinx)〗^2 +h^2/(〖4(cosx)〗^2 〖(sinx)〗^2)-(h^2 cosx)/cos〖x 〖(sinx)〗^2 〗

m^2=h^2/(4〖(cosx)〗^2 〖(sinx)〗^2), m=h/(2 cos〖x sinx 〗),

or m=h/sin2x . From: sin〖2x=h/m.〗

equilateral triangle ABC is twice the base angle bisector. Find the angles of the triangle.

AE=2BD (5)

We have. In this case, we have defined the height BD and the bisector AEby . If we take side AB as C and angle CAB as A, then ABDfrom the triangle

AE/c=sinB/sin〖3A/2〗

(∠BEA Since it lies outside the triangle , AEC∠ C+(∠A)/2=∠A+(∠A)/2=3/2∠A).and ∠ B=180^0-2∠A

AE=〖c sin〗2A/sin〖3A/2〗 (6)

will be.

By putting (4) and (5) into (1)

2sin 〖A=sin 2A/sin 〖3A/2〗 〗

is taken.

From this

cos 〖A=sin 〖3A/2〗 〗

is , which gives 3A/2=90^0+A (7) or 3A/2=99^0-A (8). A=180^0is taken from (7). This is not possible. So 3A/2=90^0-Ais , which 〖∠A=36〗^0gives

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