EUCLID'S "ELEMENTS" WORK, HIS ADVANTAGES AND DISADVANTAGES

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Annotation: In the article The advantages and disadvantages of Euclid's 13-book work "Fundamentals" are studied. These books contain an algorithm for finding the greatest common divisor of two integers and prove that there are infinitely many prime numbers. In the process of teaching high school geometry, it is necessary to rely on different axiom systems to use different textbooks. Therefore, it is extremely important for young teachers to have extensive knowledge of the requirements for the axiomatics of geometry, as well as the ability to use several different axiom systems.

Key words: diameter, inscribed angle, right angle, triangle, conditions of equality of triangles, sides of a triangle, identity, quadratic equation, circle diameter, vertical angle, polygon, straight line, surface, postulate, axiom.

The Greek scientist Euclid solved the problem of constructing geometry on a deductive principle in a way that was satisfactory for his time, writing his 13-book work "Fundamentals."

Euclid lived around 300 BC, and we do not know much about him. During the reign of Ptolemy, he taught mathematics in Alexandria and created the mathematics department of the museum founded by the king. It is said that one day the king summoned Euclid and asked, “Is there a shorter way to learn geometry than the Elements?” Euclid proudly replied, “There is no special path in geometry for kings.” In addition, Euclid’s Optics and other works are also known. The Elements have been reprinted more than 300 times since 1482 and have been translated into many languages. Let us briefly discuss the Elements.

I. The work includes the conditions for the equality of triangles, the relationship between the sides and angles of a triangle, the parallelism and perpendicularity of straight lines forming triangles, the faces of a parallelogram and a triangle, and the Pythagorean theorem.

II. In the work $(a+b)^2 = a^2 + 2ab + b^2$, $(a-b)b = ab - b^2$ and similar facts are interpreted geometrically. This book concludes with a geometric solution of the quadratic equation.

III. The book is devoted to the circle. It mainly deals with the tangents, tangents, central angles, and inscribed angles of a circle.

IV. The book shows how to construct regular rectangles, pentagons, hexagons, and 15-angled polygons by considering polygons drawn internally and externally to a circle.

V. The book is mainly devoted to the theory of proportions.

VI. The book presents the theory of similarity of polygons and finding the faces of polygons as an application of the theory of proportions.

VII-IX are devoted to arithmetic and number theory.

It is noteworthy that these books provide an algorithm for finding the greatest common divisor of two integers and prove that there are infinitely many prime numbers.

X. The book discusses the theory of irrational quantities.

XI-XII. The books are devoted to stereometry, in which polyhedra, solids of revolution and their volumes are considered, and information about regular polyhedra is given. Each of the 13 books cited begins with definitions of concepts, for example, in Book I there are 23 definitions, some of which we will cite.

1. The point is, it doesn't have any parts.
2. A line is a length without a width.
3. The boundaries of a line are points.
4. A straight line is a line that is equidistant from all its points.
5. A surface is something that has length and width.
6. The boundaries of a surface are lines.
7. A plane is a surface that is equidistant from all straight lines on it.
8. A straight angle is the angle between two lines that intersect and lie in the same plane, but do not lie on the same straight line.

After the definitions, postulates (currently, postulates and axioms are not distinguished from each other) and axioms are given.

POSTULATES

I. Let it be possible to draw a straight line from each point to any point.

II. Let each bounded straight line be able to be continued as far as desired.

III. Let it be possible to draw a circle with any radius from any center.

IV. Let all right angles be equal to each other.

V. forms interior angles $2d$with them whose sum is less than, then when they $2d$are extended in the direction of the smaller sum, they intersect on that side. This last postulate is Euclid's famous fifth postulate about parallels.

AXIOMS

I. Quantities equal to a third quantity are mutually equal.

II. If equal quantities are added together, their sums will also be equal.

III. If equal amounts are subtracted from equal amounts, the remainders will also be equal, and so on.

Another important historical significance of the "Fundamentals" is that it brought the idea of a serious logical explanation of geometry to our time. Among the great figures in the history of science up to our time, Copernicus, Galileo, Lobachevsky, Al-Khwarizmi, Beruni, Ibn Sina, Ulugbek, Omar Khayyam and others also studied mathematics from Euclid's "Fundamentals". But this work is not without its shortcomings. What are the main shortcomings of the "Fundamentals"?

1. Some of the definitions given by Euclid do not define anything, and Euclid himself does not use these definitions. The definitions contain concepts that should be defined, such as “length”, “width”, “boundary”, etc. But his definitions of the circle, triangle, right angle, obtuse and acute angle are satisfactory.

2. Euclid called some sentences postulates and others axioms. There is no logical difference between these two concepts. According to some people, he considered only sentences that define the properties of geometric figures as postulates, and the rest were taken as sentences that define the properties of any quantities.

In modern literature, axiom and postulate are used interchangeably. Similarly, axioms of order and action are also lacking.

When looking at the "Fundamentals" from a critical perspective, it is also important to note that its main shortcomings were revealed only at the end of the 19th century.

In the history of geometry, Euclid's fifth postulate plays a very important role. This postulate has attracted the attention of mathematicians since ancient times. They tried to prove this postulate, deriving it from previous postulates and axioms, but most of these scientists took some sentence that logically follows from the postulates and axioms and then claimed to have proven the fifth postulate. We will highlight the work of some of these scientists.

1. Posidonius, who lived in the 1st century BC, succeeds in proving the fifth postulate by accepting without proof the sentence "The locus of points lying on the same side and at the same distance from a straight line in a plane is a straight line."

2. The statement of the Greek mathematician Proclus (410 – 485) “The distance between two non-intersecting straight lines is finite” is equivalent to the fifth postulate.

3. The Azerbaijani scientist Nasriddin Tusi (1201 – 1274) based his argument on the following: “If $a, \ b$the first of two straight lines $AB$is perpendicular to the intersection ($A \in a, \ \ B \in b$)and the second is oblique, then $b$the side $b$of the perpendicular drawn $AB$from the straight line to the straight line $a$that forms an acute angle with $AB$is less than, $b$and the side that forms an obtuse angle with $AB$is greater than.” Based on this hypothesis, he gives his “proof” of the fifth postulate.

4. John Wallis (1616 – 1703), an English mathematician and professor at Oxford University, “proves” the fifth postulate by assuming that “there are two triangles that are similar but not equal.”

5. Hungarian mathematician Farkas Bol'yan (1775 – 1856) "proved" the fifth postulate based on the hypothesis that "any three points not lying on the same straight line lie on the same circle" or that a circle can be drawn through three such points, etc.

First, let's present a few facts whose evidence is not based on this postulate.

a) From a point outside a straight line on a plane, a straight line parallel to that straight line can be drawn.

b) An exterior angle of a triangle is greater than any of its non-adjacent interior angles.

c) Two straight lines that are perpendicular to one straight line are parallel to each other.

d) If two straight lines intersect at a straight line and the sum of the interior angles formed at the intersection $180^0$is equal to , then these straight lines are parallel.

e) If the corresponding angles of two straight lines are equal when they are intersected by a third straight line, then these straight lines are parallel, and so on.

Theorem 1.1. The hypothesis that "through a point in a plane that is not on a straight line, there passes only one straight line parallel to that straight line" is equivalent to the fifth postulate.

Theorem 1.2. The statement "The sum of the interior angles of a triangle $180^0$is equal to" is equivalent to the fifth postulate.

In addition to these, the sentences accepted by Posidonius, Proclus, Nasriddin Tusi, Vallis, and Bol'yan are also equivalent to the fifth postulate.

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