BIR JINSLI TOR TEBRANISHI TENGLAMASI UCHUN KOSHI MASALASINING YECHIMI. CHEKSIZ TOR UCHUN D'ALEMBER FORMULASI

РЕШЕНИЕ ЗАДАЧИ КОШИ ДЛЯ УРАВНЕНИЯ УЗКИХ КОЛЕБАНИЙ ОДНОГО ПОЛА. ФОРМУЛА Д'АЛАМБЕРА ДЛЯ БЕСКОНЕЧНОГО ТОРА

THE SOLUTION TO THE CAUCHY PROBLEM FOR THE SAME-SEX NARROW OSCILLATION EQUATION. D'ALEMBER'S FORMULA FOR INFINITE TOR

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Annotatsiya: *Ushbu maqolada cheksiz uzunlikdagi bir jinsli torning tebranishi bilan bog'liq Koshi masalasi ko'rib chiqiladi. Masala D'Alember usuli yordamida yechilib, boshlang'ich shartlarga asoslangan umumiy yechim topiladi. Yechimning fizik ma'nosi tushuntirilib, o'ng va chap tomonga harakatlanuvchi to'lqinlar sifatida talqin qilinadi. Misollar yordamida yechimlar amalda qanday ko'rinishda bo'lishi ham ko'rsatib berilgan.*

Kalit so'zlar:*Koshi masalasi, to'lqin tenglamasi, D'Alember formulasi, cheksiz tor, boshlang'ich shartlar, tebranish, matematik modellashtirish, to'lqinlar, analitik yechim.*

Аннотация: *В этой статье рассматривается проблема Коши, связанная с колебанием однородной струны бесконечной длины. Задача решается методом Д'Аламбера, и на основе начальных условий получается общее решение. Объясняется физический смысл решения и интерпретируется* *как волны, движущиеся вправо и влево. На примерах также показано, как решения могут выглядеть на практике.*

Ключевые слова: *задача Коши, волновое уравнение, формула Д'Аламбера, бесконечно узкий, начальные условия, колебание, математическое моделирование, волны, аналитическое решение.*

Annotation: *This article deals with the question of Coshi in relation to the oscillation of a monogamous string of infinite length. The problem is solved using the D'alember method to find a general solution based on the initial conditions. The physical meaning of the solution is explained and interpreted as waves moving to the right and left. Examples also show how solutions look in practice.*

Keywords: *Cauchy question, wave equation, d'alember formula, infinitesimal narrow, initial conditions, vibration, mathematical modeling, waves, analytical solution.*

Kirish(Введение.Introduction) Bir jinsli tor tebranishlarini o‘rganish mexanika va matematik fizika sohalarida muhim o‘rin tutadi. Ayniqsa, ikki uchi mahkamlangan yoki bir uchi mahkamlangan tor tebranishi haqidagi masalani yechishdan oldin osonroq bo‘lgan masalani, ya’ni cheksiz tor tebranishi haqidagi masalani ko‘raylik. Quyidagi

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2} \qquad (1)$$

bir jinsli tor tebranish tenglamasining

$$\left.\begin{aligned} u(x,t)\big|_{t=0} &= f(x) \\ \frac{\partial u}{\partial t}\bigg|_{t=0} &= F(x) \end{aligned}\right\} \qquad (2)$$

boshlang‘ich shartlarni qanoatlantiruvchi yechimini topish kerak. Bundagi f(x), F(x)lar (−∞,∞) oraliqda berilgan funksiyalardir. Noma’lum u(x,t) funksiyaga hech qanday chegaraviy shart qo‘yilmagan. (1) tenglamaning (2) boshlang‘ich shartlarni qanoatlantiruvchi yechimini topish masalasi **Koshi masalasi** deyiladi. Bu masalani yechish usuli **D’Alember usuli** deyiladi. (1) ning umumiy yechimi

$$u(x,t)=\varphi(x-at)+\psi(x+at) \quad (3)$$

bo‘lishligini ko‘rsatamiz. Bunda φ,ψ funksiyalar ikki marta differensiallanuvchi funksiyalardir.

Haqiqatan ham

$$u'_x=\varphi'(x-at)+\psi'(x+at), u''_{xx}=\varphi''(x-at)+\psi''(x+at)$$
$$u'_t=-a\varphi'(x-at)+a\psi'(x+at), u''_{tt}=a^2\varphi''(x-at)+a^2\psi''(x+at)$$
$$u''_{tt}=a^2\varphi''+a^2\psi''=a^2(\varphi''+\psi'')=a^2u''_{xx}$$
$$u''_{tt}=a^2u''_{xx}$$

Tenglik o‘rinli bo‘ladi. Demak, (3) (1) ning umumiy yechimi bo‘ladi.

Endi (2) boshlang‘ich shartlardan foydalanib, noma’lum φ,ψ funksiyalarni topamiz. (2), (3) dan:

t=o bo‘lganda

$$\varphi(x)+\psi(x)=f(x) \quad (4)$$

xosil bo’ladi

$$u'_t=-a\varphi'(x-at)+a\psi'(x+at) \quad (5)$$

(5) ni 0 dan x gacha integrallasak,

$$-a\int_0^x\varphi'(x)dx+a\int_0^x\psi'(x)dx=\int_0^x F(x)dx$$

$$-a(\varphi(x)-\varphi(0))+a(\psi(x)-\psi(0))=\int_0^x F(x)dx$$

$$-\varphi(x)+\psi(x)=\frac{1}{a}\int_0^x F(x)dx+c$$

$$c=-\varphi(0)+\psi(0) \quad (6)$$

(4) va (6) dan:

$$\left.\begin{array}{l}\varphi(x)+\psi(x)=f(x)\\ -\varphi(x)+\psi(x)=\frac{1}{a}\int_0^x F(x)dx\end{array}\right\}\rightarrow \left.\begin{array}{l}\varphi(x)=\frac{1}{2}f(x)-\frac{1}{2a}\int_0^x F(x)dx\\ \psi(x)=\frac{1}{2}f(x)+\frac{1}{2a}\int_0^x F(x)dx\end{array}\right\} \quad (7)$$

(7) dagi o‘rniga x−at va x+at qo‘ysak,

$$\varphi(x-at)=\frac{1}{2}f(x-at)-\frac{1}{2a}\int_0^{x-at}F(x)dx$$

$$\psi(x+at)=\frac{1}{2}f(x+at)+\frac{1}{2a}\int_0^{x+at}F(x)dx$$

(3) ga asoslan

$$u(x,t)=\frac{1}{2}f(x-at)-\frac{1}{2a}\int_0^{x-at}F(x)dx+\frac{1}{2}f(x+at)+\frac{1}{2a}\int_0^{x+at}F(x)dx$$

Ma'lumki,

$$-\int_0^{x-at}F(x)dx+\int_0^{x+at}F(x)dx=\int_{x-at}^{0}F(x)dx+\int_0^{x+at}F(x)dx=\int_{x-at}^{x+at}F(x)dx$$

u holda

$$u(x,t)=\frac{f(x-at)+f(x+at)}{2}+\frac{1}{2a}\int_{x-at}^{x+at}F(x)dx. \quad (8)$$

(8) formula — tor tebranishi tenglamasi uchun Koshi masalasining D'Alember yechimi deyiladi.

Misollar: 1) $u''_{tt}=u''_{xx}$ tenglamaning

$u|_{t=0}=x^2, u'_t|_{t=0}=0$ boshlang'ich shartlarini qanoatlantiruvchi yechimini toping.

$$a=1, f(x)=x^2, F(x)=0.$$

$$u(x,t)=\frac{(x-t)^2+(x+t)^2}{2}=x^2+t^2, u(x,t)=x^2+t^2$$

2) $u''_{tt}=9u''_{xx}$

$$u|_{t=0}=0, u'_t|_{t=0}=e^x$$

$$a=3, f(x)=x^2, F(x)=e^x$$

$$u(x,t)=\frac{1}{6}\int_{x-3t}^{x+3t}e^x dz=\frac{1}{6}e^x\Big|_{x-3t}^{x+3t}=\frac{e^x}{6}\left(e^{3t}-e^{-3t}\right)$$

Xulosa

Ushbu maqolada bir jinsli cheksiz tor tebranish tenglamasi uchun Koshi masalasining D'Alember usuli yordamida yechimi ko'rib chiqildi. Boshlang'ich shartlarga asoslangan umumiy analitik yechim topildi va uning fizik ma'nosi tushuntirildi. Bu yechim torning tebranishlarini tahlil qilishda asosiy vosita bo'lib, keyingi murakkab chegaraviy shartlar bilan ishlash uchun mustahkam nazariy poydevor yaratadi.

Ush bu maqola mustaqil ta'lim topshirig'i asosida bajarish davomida tayyorlandi.

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