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ECONOMIC PROBLEMS INTO LINEAR PROGRAMMING PROBLEMS AND SOLVING THE SIMPLEX METHOD

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As is known, problems related to the theory and application of quantitative methods and models are solved by bringing them to the problem of linear programming. The condition of certainty is understood as a situation in which all parameters and conditions of system control are certain, that is, there is no random effect. In such problems, the method of linear optimization is used, which aims to create an optimal production plan, determine the optimal volume of trade, purchase or transportation, optimal financial planning, and similar goals. Planning is one of the main functions of management.

Annotation. *This article deals with bringing economic problems to the problem of linear programming and solving them graphically. Also, at first, linear programming will be touched upon, its solution methods and mathematical interpretation of the given problem: a special emphasis will be placed on solving it through a linear function. As an example, the issue of optimal production planning for the "Olmos" furniture factory is given.*

Key words: *Multi-argument linear function, argument, linear constraints, extremum, graphical method, mathematical modeling, objective function.*

If the number of variables in the mathematical model of a linear programming problem is more than two (with some exceptions), the problem cannot be solved graphically. The simplex method is used to solve such problems.

The simplex method is a method of successively moving from one basic solution (one end of the solution polygon) to another until the objective function of a linear programming problem takes the optimal (maximum or minimum) value. This

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method is a universal method that allows you to solve any linear programming problem, unlike the graphical method, which is designed to solve problems with only two variables. is considered .

The simplex method was proposed in 1947 by the American mathematician R. Danzig, and since then it has been widely used in industrial production to solve linear programming problems involving thousands of variables and constraints. Before describing the simplex method, let us recall some concepts of a system of linear equations.

To us n variable m Given a system of equations:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n = b_m \end{cases} \qquad (1)$$

In linear programming problems $A = (a_{ij})$ $(i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n)$ matrix color $r = m$ is, $m < n$ The situation is interesting.

If (1) the system m If the determinant of the matrix formed from the coefficients before the variables is different from zero, then the basis for such variables is are called variables .

remaining $n - m$ variables are either independent or non-independent. are called variables .

If (1) the system $(x_1, x_2, \ldots, x_n)$ solutions $x_j \ge 0$ $(j = 1, 2, \ldots, n)$ If the condition is satisfied, such solutions are called feasible solutions, otherwise they are called impossible solutions .

A solution to a system in which the non-basic variables are zero is called a basic solution .

Analysis and results

For convenience in calculations, it is advisable to present the simplex method in tabular form.

Simplex table. The following algorithm is used to solve linear programming problems using the simplex method.

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Step 1. Construct an initial simplex tableau;

Step 2. Check the solution for optimality. End the process when an optimal solution is found;

Step 3. Find the state that leads to optimality;

Step 4. Switch to a new solution and return to step 2.

The general form of a simplex table is given in Table 1 (m – number of conditions, n – number of variables)

<table>
<tr><td rowspan="2"></td><td rowspan="2"></td><td rowspan="2"></td><td colspan="8">Objective function</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Basis changers</td><td>Coefficients of the objective function</td><td>Basis solution values</td><td colspan="8">Coefficients of the terms of the problem</td></tr>
<tr><td colspan="2">F_j</td><td></td><td colspan="8">$C_j - F_j$ line definition</td></tr>
<tr><td colspan="2">$C_j - F_j$</td><td></td><td colspan="8">A string defining the optimality criterion</td></tr>
</table>

Table 1. Simplex table view

• The first row of the table records all (main and additional) variables;

• Table B The first column, separated by the letter , lists the basic variables.

• The second row of the table, starting from cell 3, lists the coefficients of the objective function.

• C_b The coefficients of the variables included in the basis are placed in the column (except for the last two rows).

• The coefficients of the conditions are given in the rows dedicated to the basic variables.

• P_0 The column contains the values of the basic variables.

• Last $C_j - F_j$ The line is aimed at determining the optimality criterion.

• F_j using the latest information $C_j - F_j$ is a row, the last cell of which contains the current value of the objective function.

This

$$\begin{cases} x_1 + 2x_2 \le 10 \\ 2x_1 + x_2 \le 14 \end{cases}$$

$$x_1 \ge 0, \quad x_2 \ge 0$$

$$F = 2x_1 + 3x_2 \to \max$$

We determine the solution of the problem using the simplex method. To bring the problem into canonical form, we use the following addition s_1, s_2 We introduce variables:

$$\begin{cases} x_1 + 2x_2 + s_1 = 10 \\ 2x_1 + x_2 + s_2 = 14 \end{cases}$$

$$x_1 \ge 0, \quad x_2 \ge 0, \quad s_1 \ge 0, \quad s_2 \ge 0 \qquad (2)$$

$$F = 2x_1 + 3x_2 + 0 \cdot s_1 + 0 \cdot s_2 \to \max$$

There are a total of 4 equations in the system of equations, i.e. 2 basic ones x_1, x_2 and 2 additional ones s_1, s_2 There are variables. The vector of coefficients of the objective function is C, and the matrix of coefficients of the constraint conditions is A and the right-hand side vectors of the conditions B are defined as follows:

$$C = (c_1; c_2; c_3; c_4) = (2; 3; 0; 0)$$

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \end{pmatrix}, \qquad B = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} 10 \\ 14 \end{pmatrix}$$

Step 1. Construct an initial simplex tableau

Let's construct a simplex table for our example above. The objective function

$$z = 2x_1 + 3x_2 + 0 \cdot s_1 + 0 \cdot s_2$$

Write it in the form of a table, taking into account the system (2)

We fill in the following (Table 2).

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B	C_b	P_0	x_1	x_2	s_1	s_2
			2	**3**	**0**	**0**
s_1	**0**	**10**	**1**	**2**	**1**	**0**
s_1	**0**	**14**	**2**	**1**	**0**	**1**
F_j		**0**	**0**	**0**	**0**	**0**
$C_j - F_j$			**2**	**3**	**0**	**0**

Table 2. Elementary simplex table

2, 3, and 4 of the initial simplex tableau consist directly of the objective function and the system coefficients (A matrix, C and B pay attention to vectors). F_j The row elements are found as follows. The vector consisting of the coefficients of the objective function in the basis is scalar multiplied by the vectors in the condition column. That is, $C_b = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ a vector $A_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ is scalar multiplied by a vector, etc. In this way F_j all elements of the row are found. $C_j - F_j$ The row elements are the coefficients of the objective function, respectively F_j is obtained by subtracting the elements of the row. Since the variables not included in the basis are equal to zero $x_1 = 0$, $x_2 = 0$.the value of the basis variables is taken from the last column: $s_1 = 10$, $s_2 = 14$. F_j The number in the last cell of the row is the value of the objective function at the initial step. $F = 0$.

In the first step, the values of the last row match the coefficients of the objective function. This completes the first step.

Step Two. Check the result for optimality

Optimality of the obtained result $C_j - F_j$ is determined by the non-negativity of all numbers in the row. If $C_j - F_j$ all elements in the row are zero or negative If the result obtained is optimal and the process is completed. If there is at least one positive element among these elements, then optimality has not been achieved and the solution can be improved.

In our example, the result is not optimal because the last row contains two positive numbers. This completes the verification of the optimality condition.

Step Three. Finding the optimality-directing state

We determine the maximum element from the last row of the initial table, which is equal to 3. The column containing the largest positive element in the last row of the simplex table is called the decisive column (Table 3).

B	C_b	P_0	x_1	x_2	s_1	s_2	P_0 / a_{ij}
			2	**3**	**0**	**0**	
s_1	**0**	**10**	**1**	**2**	**1**	**0**	**10/2=5**
s_1	**0**	**14**	**2**	**1**	**0**	**1**	**14/1=14**
F_j		**0**	**0**	**0**	**0**	**0**	
$C_j - F_j$			**2**	**3**	**0**	**0**	

Table 3. Determination of the decisive element

In the table given in Table 3, the decisive column is indicated by an arrow. In order to find the decisive row, we introduce an additional column and divide the elements of the column by the elements of the decisive column. We take the smaller of the resulting numbers: $\min\{5, 14\} = 5$. Therefore, the third row of the table is the decisive row, and this row is indicated by an arrow. The element located at the intersection of the decisive row and the decisive columns is the decisive is called the element . In our example, the decisive element is equal to 2 and is shown in red in the table. This completes step 3.

Step 4. Switch to a new solution

The transition to a new solution begins with the exchange of basic variables. The basic variable at the beginning of the solution row is exchanged with the variable in the solution column, and the corresponding coefficients are also exchanged. The elements in rows 3 and 4 of the simplex table are recalculated using the Gauss-Jordan method using the solution element. The Gauss-Jordan method proceeds as follows:

1) The decisive row is divided by the decisive element. The remaining elements of the decisive column are filled with zeros.

2) The remaining rows are recalculated using the "rectangle" method. You are familiar with this method from the topic of solving a system of linear equations using the Gauss-Jordan method.

Let's mention the "rectangle" method. $a(i,\ j) = a_{ij}$ Let us define the element at the intersection of $a(s, k)$ the -row and -column j of a table such as . Let i – be the decisive element and $a(i,\ j)$ the element to be recalculated. The table $a(s, k)$ and $a(i,\ j)$ Using the cells containing the values, we can construct a right rectangle as shown in Table 4 below.

$a(i,\ j)$	$\ldots$	$a(i, k)$
$\ldots$		$\ldots$
$a(s,\ j)$	$\ldots$	$a(s, k)$

Table 4. Rectangle method

$a(i,\ j)$ new value of $a^*(i,\ j)$ is calculated using the following formula:

$$a^*(i,\ j) = a(i,\ j) - \frac{a(s,\ j)\cdot a(i, k)}{a(s, k)}$$

As a result of the recalculation, we arrive at the following table 5. Thus, a second table was constructed, and a new simplex table was created.

To speed up calculations, it is advisable to use the following rules.

B	C_b	P_0	x_1	x_2	s_1	s_2
			2	**3**	**0**	**0**
x_2	**3**	**5**	**1/2**	**1**	**1/2**	**0**
s_1	**0**	**9**	**3/2**	**0**	**–1/2**	**1**
F_j		**15**	**3/2**	**3**	**3/2**	**0**
$C_j - F_j$			**1/2**	**0**	**–3/2**	**0**

Table 5. Second simplex table

• If the decisive row contains elements equal to 0, the value of the corresponding column elements remains unchanged in the new table;

• If the decisive column contains elements equal to 0, the corresponding row in the new table remains unchanged. We proceed to the second step and check the optimality criterion again.

The last row of the new simplex table contains a positive element $\frac{1}{2}$ Since there is no optimal solution, we will construct a new table. Now the decisive column x_1 is the one corresponding to the only positive element in the last row (Table 6).

B	C_b	P_0	x_1	x_2	s_1	s_2	P_0 / a_{ij}
			2	**3**	**0**	**0**	
x_2	**3**	**5**	**1/2**	**1**	**1/2**	**0**	**10**
s_1	**0**	**9**	**3/2**	**0**	**–1/2**	**1**	**6**
F_j		**15**	**3/2**	**3**	**3/2**	**0**	
$C_j - F_j$			**1/2**	**0**	**–3/2**	**0**	

Table 6. Determination of the decisive element

Minimum value in the last column $\min\{10, 6\} = 6$ Since the decisive line is the fourth line. So, x_1 The variable enters the basis, s_2 and comes out of the basis. We construct a new table according to the above rule (Table 7).

B	C_b	P_0	x_1	x_2	s_1	s_2
			2	**3**	**0**	**0**
x_2	**3**	**2**	**0**	**1**	**2/3**	**–1/3**
x_1	**2**	**6**	**1**	**0**	**–1/3**	**2/3**
F_j		**18**	**2**	**3**	**4/3**	**1/3**
$C_j - F_j$			**0**	**0**	**–4/3**	**–1/3**

Table 7. The last simplex table

If we look at the last row of the resulting table, all the elements in the row are non-negative, which indicates that we have reached the optimal solution. Optimal plan from the table $x_1 = 6$, $x_2 = 2$, $s_1 = 0$ and $s_2 = 0$ and the optimal value of the objective function $F_{max} = F(6;\, 2) = 2 \cdot 6 + 3 \cdot 2 = 18$ In the last table, the optimal value of the objective function is formed in the pink cell.

Conclusion

Solving a linear programming problem using the simplex method is a universal method, in which there is no limit to the number of variables, as in the graphical method. In addition, the algorithm for solving a linear programming problem using the simplex method is available in many application programs. In particular, linear programming problems can be solved using the simplex method using MS Excel and POM QM for Windows .

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