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SOME APPLICATIONS OF THE PYTHAGOREAN THEOREM

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ABSTRACT

The Pythagoraean Theorem has been the driving force for the mathematical world since its discovery. It is generally credited to Pythagoras, a Greek mathematician even though archaeological evidence has stated that Babylonians, Egyptians, and Chinese people discovered it before him. It is because he was the first who revealed this theorem to the world by stating it clearly. The Pythagoras Theorem has many practical aspects as it has helped develop many tools and machinery, all of which have in some form shaped this developing world of ours. The theorem, which deals with the right-angled triangle, has also been used as a crucial base for many more laws. In this research paper, I have written many distinct ways, which we can use to prove the Pythagorean Theorem. These ways include the infamous Windmill proof (also known as bride's chair proof) which was discovered by Euclid (another great Greek mathematician who has greatly helped the geometric world), the proof where we draw an altitude and use properties of similarity also called proof by similar triangles, Garfield's proof which was discovered by James Garfield, the 20th US president and the proof using differentials which were earlier arduous to prove through.

Like any other triangle, there are 3 sides in a right-angled triangle. The longest side in this type of triangle is called the hypotenuse or "C" which is always opposite to the largest angle or in this case: 90 degrees. The other two sides are termed "A" and "B". These 3 sides together constitute the basic elements of the Pythagorean Theorem.

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The Pythagoras theorem states that "In a right-angled triangle, the square of the hypotenuse side is equal to the sum of squares of the other two sides" Or we can say: A2+B2 = C2

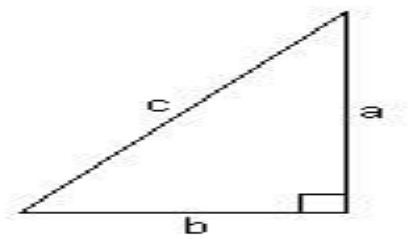
C = whole square root of A2+B2

B = whole square root of C2 - A2

A = whole square root of C2 - B2

Right angled = 90 degrees

Hypotenuse = Side opposite to the right angle



The theorem is named after the Greek mathematician and philosopher, Pythagoras who was the one who made it well-known around the world. Although there has been archeological evidence that math's most famous theorem was first found by the Babylonians and the Egyptians, its discovery was not as widespread and was eventually lost from people's minds. *Different Ways to prove*- 1) The Brides chair or Euclid I 47

2) Pythagorean Theorem proof using similar triangles
3) Garfield's proof of the Pythagorean Theorem
4) Proof using differentials

The Pythagorean Theorem is one of mathematics's most elementary yet crucial formulas which has helped the modern world grow for the last 2500 years. It has greatly helped in studying the length measures and establishing the relationship between the three sides of a right-angled triangle. Besides that, it has also further advanced trigonometry and has been used in various fields like construction, architecture, oceanography, aerospace, navigation, and more.

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Its application is important as right angles are universal. Right angles are present in a graph with multiple axes, a table, a building, and even inside the structure of an atom.It has many real life applications like:-

1) Finding the gradient of a particular landscape and height of a point in the sky from an observer on the ground.

Painting on a Wall: Painters use ladders to paint on high buildings and complete their work with the help of

Pythagoras' theorem by determining how tall a ladder needs to be to safely place the base away from the wall so it won't cause an accident.

2) Whales are endangered from all fronts such as climate change, whaling, toxins in water and even being struck by ships! To avoid hitting the whales, the sailors use the theorem, by pinpointing the exact location of whales using their sounds when whales sing. We can also use Active SONAR to find the location of other fishes, but in the case of whales, they hate that sound so we use their natural sounds.

3) What Size Television Should You Buy: T.V. size is always given in terms of diagonal. Suppose a T.V. size is given to be 32 inches, then its size is the value of the diagonal or hypotenuse.

4) Finding the Right Sized Computer: Monitor size is always given a measure of diagonal.

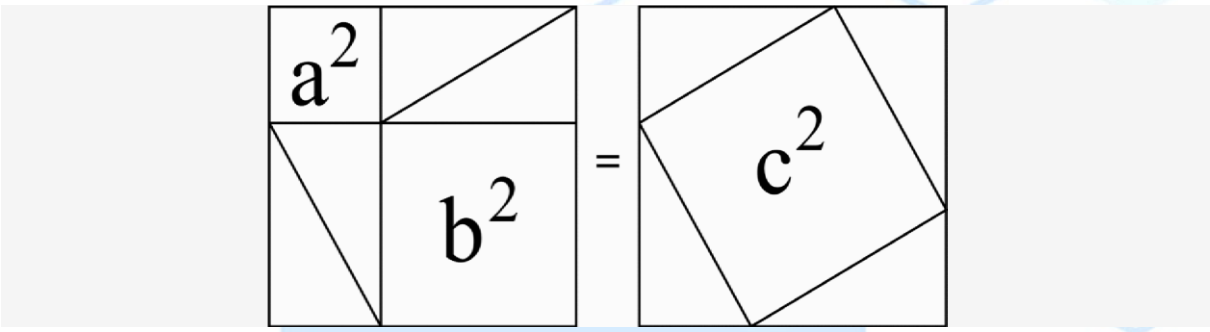
5) An aerospace engineer can also use the Pythagorean Theorem when creating a missile or a rocket. The tip of the missile makes an isosceles triangle which can make two right triangles.

The theorem has been used centuries before Pythagoras with more and more archaeologists finding different tablets and books implicitly using it. For example, a few Babylonian tablets (circa 1900–1600 BCE) indicate knowledge of the theorem to some extent, as they had accurately calculated the square root of 2 which can be shown by the Pythagorean Theorem(In a right-angled triangle if A and B each have a unit size measurement, length of C will be root 2). It is mentioned in the Baudhayana sulba-sutra from India, which was written between 800 and 400 BCE.

We are not even sure if Pythagoras discovered the theorem himself!

Pythagoras had traveled many places like Egypt, Babylon, and India before soon settling in Croton (now Crotone, Italy) and set up a school, or in modern terms, a monastery, where all members took strict vows of secrecy, and all new mathematical results for several centuries were attributed to his name. Thus, there is some doubt whether Pythagoras himself proved the theorem that bears his name.

Euclid validated the theorem by his famous bride's chair proof in Euclid's Elements, Book 1, Proposition 47. Subsequently, more and more proofs showered the mathematical formula and today there are hundreds of different ways we can employ to prove it. A book by Elisha Scott Loomis called The Pythagorean Proposition alone contains an impressively distinct 367 techniques for proving the most well-known theorem.



A visual illustration used to prove the Pythagorean Theorem by rearrangement.

There are 2 squares each with 4 identical triangles, which are only arranged in a different manner. Thus, the area excluding these triangles must be equal. Or we can say a2 + b2 =c2.

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Thus, proving the Pythagorean Theorem in which the square of the hypotenuse is equal to the sum of the square of the other two sides.

Euclid was the first person to prove the theorem in around 300 B.C.

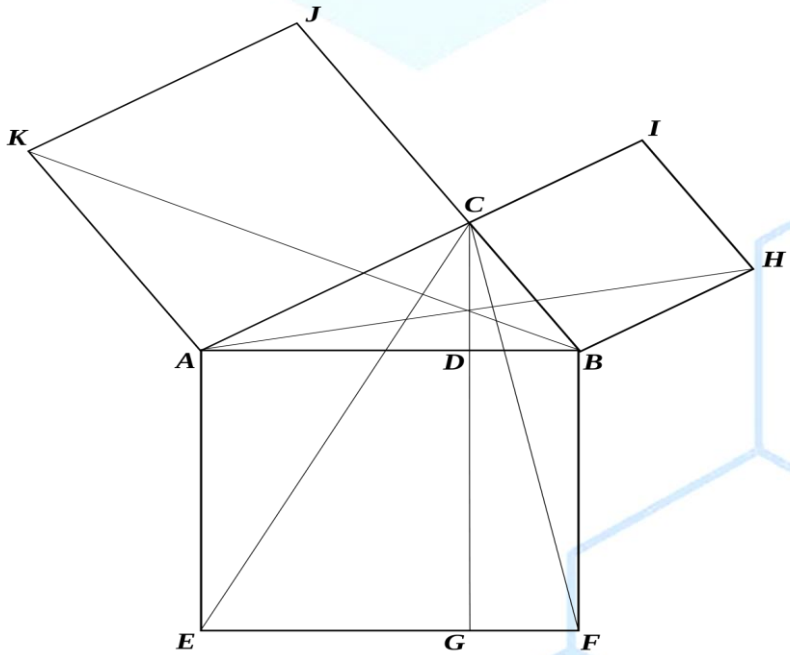
If you take a triangle ABC with angle C as 90 degrees, the hypotenuse will be AB.

We draw 3 squares while taking the 3 sides of the triangle ABC as base sides to get square ACJK (with base AC), BCIH (with base BC), and ABFE (with base AB).

We already know that the area of a square is equal to the side raised to power 2.

So the area of square ACJK is AC2, BCIH is BC2, ABFE is AB2.

We can see the sum of the area of the square ACJK and BCIH as equal to ABFE. Thus AB2= AC2+BC2



Pythagorean Theorem proof using similar triangles

Construction: Draw a perpendicular BD on AC

In tri ABD and tri ABC tri ABD and tri ABC we have, angle BAD = angle BAC (angle A is

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common in both triangles.) angle ABC = angle ADB = 90 degrees (by construction)

Therefore tri ABC~tri ABD (By AA similarity i.e. angle-angle similarity)

So,AD/AB=AB/AC⇒AB2 = AD×AC (1)

In tri BDC and tri ABC we have,

angle BCD = angle BCA (angle C is common in both triangles)

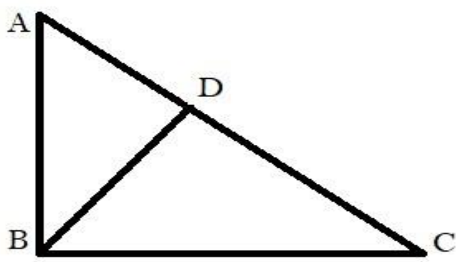
angle ABC = angle ADC = 90 degrees

Therefore tri ABC~tri BDC (By AA similarity i.e. angle-angle similarity)

So,⇒DC/BC=BC/AC⇒BC2 = AC×DC (2)

Adding equation (1) and (2) , we get

⇒AB2 + BC2 = AD×AC + AC× DC⇒AB2 + BC2 = AC(AD + DC)⇒AB2 + BC2 = AC(AC)⇒AB2 + BC2 = AC2 Hence proved.



Garfield's proof of the Pythagorean Theorem

Garfield, who was the 20th president of the states, was also the one to prove the Pythagorean Theorem in his unique way.

Construct a right triangle with the angle between A and B as the right angle. Connect the endpoints of A and B with C.

Construct a similar triangle such that side B now extends in a straight line from the original side A. Now, draw side A along the top parallel to the bottom original side B, and side C connecting the endpoints of the new A and B.

We want to know the angle between the 2 triangles. Let the angle on the far right side of the first triangle be theta such that the other angle of the same triangle becomes 90-theta.

The bottom angle in the new triangle will also be theta (because similar triangles have equal angles). Now the sum of the 3 angles will be 180 degrees (the straight line has 180 degrees) So we will get the angle between the 2 triangles as 90 degrees.

Now join side a above to side b at the bottom. This forms a trapezium and we can find its area as 1/2 x (sum of parallel sides) x (altitude).

1/2 (A+B) x (A+B)

Area of 3 triangles = 1/2 (ab) + 1/2 (c square) + 1/2 (ab) = ab + 1/2 (c square)

Area of trapezium= area of the 3 triangles

(A+B)(A+B)= 2ab + (c square)

On solving we can prove the theorem by getting:

(a square) + (b square) = (c square)

Proof using differentials

Let ABC be a right-angled triangle with BC as the hypotenuse. Also, let BC be y, AC is x, and AB is a.

If x is increased a little by dx leaning towards D, y also gets increased by dy. Together dy and dx form a triangle CDE, which is similar to ABC, with CE perpendicular to the hypotenuse BD. Hence, the ratio of their sides will be dy/dx = x/y.

On rearranging we get,

(dy)(y) = (dx)(x)

Now we integrate these

terms y2 = x2 + c

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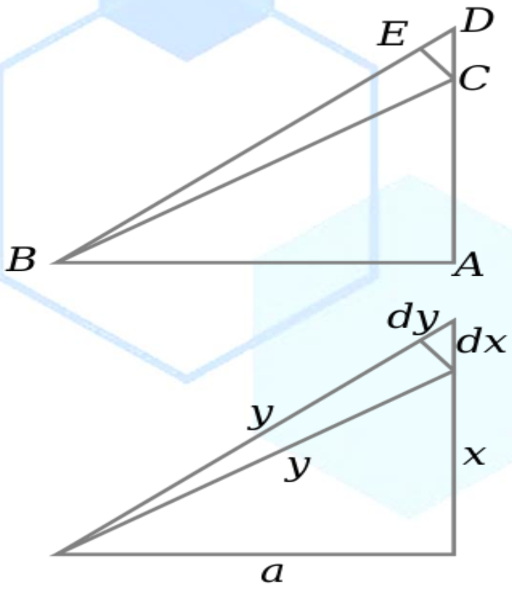
We know the constant c is equal to a2 because x = 0, y = a

Therefore, we get our beloved theorem in the form y2 = x2 + a2



Converse of the Pythagorean Theorem is also true with its statement being: if the square of any 2 sides of a triangle is equal to the square of the third side, the triangle is right angled. Pythagoras Theorem have helped us prove the existence of irrational numbers-

Earlier irrational numbers couldn't be shown in a number line. But because of the unique nature of the Pythagorean Theorem, which deals with square roots to find length, humanity was able to represent these numbers in a number line for the first time in recorded history.The Pythagoras Theorem has also acted as an important relation for other laws, and theorems all of which have diverse and wide ranging impact on the mathematical world and hence our lives. In fact, the formula of square of sin A + square of sin B = 1 was derived by this law.

The euclidean distance which is the whole root of (x1-x2) (y2-y1) or the dot product and cross product all had the Pythagorean Theorem as its base. All in all, I believe the Pythagoras Theorem has been influencing the way humans have been living since its discovery. It has helped make machines and tools many of which have

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roots derived from it. The theorem's simplicity and compact nature as well as its universality in theoretical and practical aspects of every sphere of life was a revolution to the way mankind thought and worked. Thereby, It has been historically acting as the seer of modern endeavour, helping us develop many comforts that we now enjoy.

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