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THEORETICAL BASIS AND PRACTICAL APPLICATION OF THE SEIDEL METHOD : ITERATIVE APPROACH TO SOLVING LINEAR SYSTEMS

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Abstract

In the article Seidel method , that is linear one step first orderly iterative of the method essence , application and approach conditions surrounding illuminated . Seidel method simple iteration from the method different aspects , in particular , iteration in the process initial from rapprochements come out consecutively calculation scheme The method is explained . approach condition theorem in the form of brought , its proof matrices and their typical numbers based on done increased . Seidel method approach for matrix typical by the modulus of numbers suddenly small to be necessity It is also emphasized that simple iteration method with in comparison Seidel method advantages and disadvantages , conditions compliance when done faster approach theorem and examples through is based on . Article in the end practical example brought , Seidel 5 rooms of the method in accuracy solution find process table in the form of shown . This article linear algebra and iterative methods in the field research take visitors for important source become service does .

Key words : Zeydel method , iterative method , linear algebra, approximation condition , simple iteration , matrices , eigenvalues numbers , solution accuracy , system solution , theorem proof

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Seidel method linear one step first orderly iterative This method is simple iteration method this with difference does initial according to approximation $\left(x_1^{(0)}, x_2^{(0)}, \ldots, x_n^{(0)}\right)'$ $x_1^{(1)}$ what we find . Then $\left(x_1^{(1)}, x_2^{(0)}, \ldots, x_n^{(0)}\right)'$ according to $x_2^{(1)}$ will be found and etc. All $x_i^{(1)}$ from the reflection of then $x_i^{(2)}, x_i^{(3)}, \ldots$ are found . More precisely as in other words , calculations following scheme according to take goes to :

$$x_1^{(k+1)} = \frac{b_1}{a_{11}} - \sum_{j=2}^{n} \frac{a_{1j}}{a_{11}} x_j^{(k)},$$

$$x_2^{(k+1)} = \frac{b_2}{a_{22}} - \frac{a_{21}}{a_{22}} x_1^{(k+1)} - \sum_{j=2}^{n} \frac{a_{2j}}{a_{22}} x_j^{(k)},$$

....

$$x_i^{(k+1)} = \frac{b_i}{a_{ii}} - \sum_{j=i}^{i-1} \frac{a_{ij}}{a_{ii}} x_i^{(k+1)} - \sum_{j=i+1}^{n} \frac{a_{ij}}{a_{ii}} x_i^{(k)},$$

....

$$x_n^{(k+1)} = \frac{b_n}{a_{nn}} - \sum_{j=1}^{n-1} \frac{a_{nj}}{a_{nn}} x_i^{(k+1)}.$$

Now Zeiler method approach condition see We're leaving . This condition following theorem with is given .

Theorem . Seidel method approach for

$$\begin{bmatrix} a_{11}\lambda & a_{12} & \ldots & a_{1n} \\ a_{21}\lambda & a_{22}\lambda & \ldots & a_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{n1}\lambda & a_{n2}\lambda & \ldots & a_{nn}\lambda \end{bmatrix} = 0$$

of the equation all roots modules according to alike to be necessary and enough .

$$C=\begin{pmatrix} a_{11} & 0\ldots & 0\ldots0 \\ a_{21} & a_{22} & 0\ldots0 \\ \ldots & \ldots & \ldots\ldots \\ a_{n1} & a_{n2} & a_{n,n-1}\ldots a_{nn} \end{pmatrix}, D=\begin{pmatrix} 0 & a_{12}\ldots\ldots a_{1,n-1} & a_{1n} \\ 0 & 0\ldots\ldots a_{2,n-1} & a_{2n} \\ \ldots & \ldots\ldots\ldots & \ldots\ldots \\ 0 & 0\ldots\ldots\ldots0 & 0 \end{pmatrix}$$

matrices sum $A=C+D$ in the form of writing we get . In that case $A\overline{x}=\overline{b}$ system

$$C\overline{x}=-D\overline{x}+\overline{b}$$

in the form to write possible . Seidel method and

$$C\overline{x}^{(k+1)}=-D\overline{x}^{(k)}+\overline{b}$$

in the form of from etherization consists of .This PCB $\overline{x}^{(k+1)}$ in relation to if we solve :

$$\overline{x}^{(k+1)}=-C^{-1}D\overline{x}+C^{-1}\overline{b}.$$

This is Seidel method matrix $-C^{-1}D$ was simple to iteration equal strong that shows . So by Theorem 1, Seidel method approaching to be for $-C^{-1}D$ matrix all typical numbers modules according to suddenly small to be necessary and enough . That's why also for

$$\det(\lambda E+C^{-1}D)=0$$

The equation all roots modular according to suddenly small to be necessary.If this equation roots this

$$\det(C+D)=0$$

Equation roots with overlay above to fall Let 's show , theorem proof It will be . This and as follows is shown :

$$\det(\lambda E+C^{-1}D)=\det[C^{-1}C(\lambda E+C^{-1}D]=\det[C^{-1}(\lambda C+D)]=\det C^{-1}\cdot\det(\lambda C+D).$$

Here $C^{-1}\neq 0$ that was for one kind to the roots has .

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use (8.28) $\overline{x}^{(k+1)} = \overline{B}\overline{x}^{(k)} + \overline{c}$ saying , simple iteration method with detachable If , then the process (8.28) approach for

$$\begin{pmatrix} -\tau & b_{12}\ldots & b_{1n} \\ b_{21} & -\tau\ldots & b_{2n} \\ \ldots & \ldots\ldots & \ldots \\ b_{n1} & b_{n2}\ldots & -\tau \end{pmatrix} = 0$$

The equation all $\tau_1, \tau_2, \ldots\ldots, \tau_n$ roots modules according to suddenly small to be need .

Equations by comparison Let's see , it's simple. iteration method with Seidel method approach The fields are generally different . said to the idea We are coming . It really is . systems there are , they are for simple iteration method approaches . Seidel method and moves away and on the contrary so systems to bring maybe they for Seidel method approaching is simple iteration method moves away .

Terms and conditions any one if done , simple to iteration relatively Seidel method faster This is the following in the theorem further more precisely expressed .

Theorem. If $\max_i \sum_{j=1, j\neq i}^{n} \left|\frac{a_{ij}}{a_{ii}}\right|$ any of the following conditions is met, then the Optional Initial Approximation $\overline{x}^{(0)}$ The Seidel method converges for , and this convergence is no slower than the convergence of the simple iteration method when the first condition is met.

Proof .

The following designations we enter :

$$\alpha_{ij} = -\frac{a_{ij}}{a_{ii}}, \beta_{ij} = -\frac{b_i}{a_{ii}}, \mu = \max_i \sum_{j=1, i\neq j}^{n} \left|\alpha_{ij}\right|$$

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Let's assume that the first condition let it be done then $\mu<1$ $\mu<1$ will be . In these designations Seidel method this

$$x_1^{(k+1)}=\sum_{j=1}^{n}\alpha_{ij}x_j^{(k+1)}+\sum_{j=i+1}^{n}\alpha_{ij}x_j^{(k)}+\beta_t$$

Scheme according to take From here outside of the theorem first condition when done $\bar{x}=B\bar{x}+\bar{c}$ The system is a single solution yes , this the solution for example , simple iteration can be found with . So ,

$$x_i=\sum_{j=i+1}^{n}\alpha_{ij}x_j+\beta_i$$

minus to modules let's go ,

$$\left|x_i-x_i^{(k+1)}\right|i\le\sum_{j=1}^{i-1}\left|a_{ij}\right|\left|x_j-x_j^{(k+1)}\right|+\sum_{j=i+1}^{n}\left|a_{ij}\right|\left|x_i-x_i^{(k)}\right|\le\max_j\left|x_j-x_j^{(k)}\right|\left|\sum_{j=1}^{i-1}\left|a_{ij}\right|+\max_j\left|x_j-x_j^{(k)}\right|\sum_{j=i+1}^{n}\left|a_{ij}\right|=\left\|\bar{x}-\bar{x}^{(k+1)}\right\|_1\times\right.$$
$$\times\sum_{j=1}^{i-1}\left|a_{ij}\right|\left\|\bar{x}-\bar{x}^{(k)}\right\|_1\sum_{j=i+1}^{n}\left|a_{ij}\right|$$

Coming comes out . The following

$$p_i=\sum_{j=1}^{i-1}\left|\alpha_{ij}\right|,q_i=\sum_{j=i+1}^{n}\left|\alpha_{ij}\right|$$

Marking if we enter ,

$$\left|x_s-x_s^{(k+1)}\right|=\max_i\left|x_i-x_i^{(k+1)}\right|=\left\|x-x^{(k+1)}\right\|_1$$

$$\left|x_i-x_i^{(k+1)}\right|\le p_i\left\|\bar{x}-\bar{x}^{(k+1)}\right\|_1+q_i\left\|\bar{x}-\bar{x}^{(k)}\right\|_1 \quad (8.35)$$

It will be . Let's assume , $\max_i\left|x_i-x_i^{(k+1)}\right|$ to $i=s=s(k)$ $i=s=s(k)$ when be achieved :

$$\left|x_s-x_s^{(k+1)}\right|=\max_i\left|x_i-x_i^{(k+1)}\right|=\left\|x-x^{(k+1)}\right\|_1$$

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that time , assuming that in (8.85) $i = s$,

$$\left\|\bar{x} + \bar{x}^{(k+1)}\right\|_1 \le p_s \left\|\bar{x} + \bar{x}^{(k+1)}\right\|_1 + q_s \left\|\bar{x} + \bar{x}^{(k)}\right\|_1$$

Or

$$\left\|\bar{x} + \bar{x}^{(k+1)}\right\|_1 \le \frac{q_s}{1 - p_s} \left\|\bar{x} + \bar{x}^{(k)}\right\|_1$$

To inequality has we will be . If

$$\mu_1 = \max_i \frac{q_i}{1 - p_i}$$

If we assume that

$$\left\|\bar{x} + \bar{x}^{(k+1)}\right\|_1 \le \mu_1 \left\|\bar{x} + \bar{x}^{(k)}\right\|_1$$

To inequality has we will be .

Now $\mu_1 \le \mu$ that we will show .

Indeed .

$$p_i + q_i + \sum_{j=1,\, j \ne i}^{n} \left|\alpha_{ij}\right| \le \mu < 1$$

What happened? for $q_i \le \mu - p_i$. So

$$\frac{q_i}{1 - p_i} \le \frac{\mu - p_i}{1 - p_i} \le \frac{\mu - \mu p_i}{1 - p_i} = \mu \text{ from this and } \mu_1 \le \mu$$

Coming it comes out of inequality $\left\|\bar{x} + \bar{x}^{(k+1)}\right\| \le \mu_1^{k+1} \left\|\bar{x} + \bar{x}^{(0)}\right\|_1$ what harvest We do . This is of the theorem first condition when done Seidel method that it is approaching means . inequality and Seidel method approach simple iteration to the method relatively slowly that it is not shows .

Now of the theorem second condition when done Seidel method approaching we'll see

Here $\mu' = \sum_{j=1, j\neq i}^{n} |\alpha_{ij}|$ we can say .

Let's assume , $\bar{x} = (x_1, x_2, ..., x_n)'$ and $\bar{x}^k = (x_1^k, x_2^k, ..., x_n^k)'$ suitable accordingly $\bar{x} = B\bar{x} + \bar{c}$ system solution and Seidel process k - approach Let it be . In that case

$$x_i = \sum_{j=1}^{i-1} \alpha_{ij} x_j + \sum_{j=i+1}^{n} \alpha_{ij} x_j + \beta_j$$
$$x_i^{(k+1)} = \sum_{j=1}^{i-1} \alpha_{ij} x_j^{(k+1)} + \sum_{j=i+1}^{n} \alpha_{ij} x_j^{(k)} + \beta_j$$
$$(i = 1, 2, ..., n)$$

Of these $\left|x_i - x_i^{(k+1)}\right| \leq \sum_{j=1}^{i-1} |\alpha_{ij}| \left|x_j - x_j^{(k+1)}\right| + \sum_{j=1}^{n} |\alpha_{ij}| \left|x_j - x_j^{(k)}\right|$ come It comes out . inequalities all $i = 1, 2, ..., n$ by let's collect :

$$\sum_{i=1}^{n} \left|x_i - x_i^{(k+1)}\right| \leq \sum_{i=1}^{n} \sum_{j=1}^{i-1} |\alpha_{ij}| \left|x_j - x_j^{(k+1)}\right| + \sum_{i=1}^{n} \sum_{j=i+1}^{n} |\alpha_{ij}| \left|x_j - x_j^{(k)}\right|$$

and collection condition we will change .

$$\sum_{i=1}^{n} \left|x_i - x_i^{(k+1)}\right| \leq \sum_{i=1}^{n-1} \left|x_j - x_j^{(k+1)}\right| \sum_{i=j+1}^{n} |\alpha_{ij}| + \sum_{i=1}^{n} \left|x_j - x_j^{(k)}\right| \sum_{i=n}^{j-1} |\alpha_{ij}| \quad (8.38)$$ now

$$s_j = \sum_{i=j+1}^{n} |\alpha_{ij}|$$
$$t_i = \sum_{i=n}^{j-1} |\alpha_{ij}|$$
$$(j - 1, 2, ..., n-1)$$

And

$$s_n = 0, t_n = \sum_{i=1, i\neq j}^{n} |\alpha_{ij}|$$

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We get . It is obvious . it is clear ,

$$s_j + t_i = \sum_{i=1, i\neq j}^{n} |\alpha_{ij}| = \mu' < 1$$

From this and , $s_j < 1$ come it turns out . inequality following

$$\sum_{i=1}^{n} |x_i - x_i^{(k+1)}| \leq \sum_{i=1}^{n} s_j |x_j - x_j^{(k+1)}| + \sum_{i=1}^{n} t_j |x_j - x_j^{(k)}|$$

Or

$$\sum_{i=1}^{n} (1-s_j)|x_i - x_i^{(k+1)}| \leq \sum_{i=1}^{n} t_j |x_j - x_j^{(k)}|$$

to look has It will be .

Now $t_j \leq \mu' - s_j \leq \mu' - s_j\mu' = \mu'(1-s_j)$ that it was for

$$\sum_{i=1}^{n} (1-s_j)|x_j - x_j^{(k+1)}| \leq \mu' \sum_{i=1}^{n} (1-s_j)|x_j - x_j^{(k)}| \leq (\mu')^k \sum_{i=1}^{n} (1-s_j)|x_j - x_j^{(k)}|$$ come It turns out .

From this and $\mu' < 1$ that was for

$$\lim_{k\to\infty} \sum_{i=1}^{n} (1-s_j)|x_j - x_j^{(k)}| = 0$$

Harvest will be . So ,

$$\lim_{k\to\infty} x_j^{(k)} = x_j$$
$$(j = 1, 2, \ldots, n)$$

Harvest that 's it with theorem complete complete proof was done .

Now example we'll see

Example: Seidel method with system solution 5 rooms in accuracy be found .

Solution : system in appearance writing we will get and initial approach

$\bar{x}^{(0)}$ as simple iteration as in the method $\bar{x}^{(0)} = (0,6;0,44;0,95;1;1,6)'$ Here we have iteration only one step we bring :

$$x_1^{(1)} = 0,6 - 0,1x_2^{(0)} + 0,3x_3^{(0)} + 0,2x_4^{(0)} - 0,1x_5^{(0)} = 0,6 - 0,1 \cdot 0,44 + 0,3 \times$$
$$\times 0,95 + 0,2 \cdot 1 - 0,1 \cdot 1,6 = 0,881;$$

$$x_2^{(1)} = 0,44 + 0,04x_1^{(1)} - 0,04x_3^{(0)} + 0,2x_4^{(0)} + 0,08x_5^{(0)} = 0,44 + 0,04 \cdot 0,881 -$$
$$- 0,04 \cdot 0,95 + 0,2 \cdot 1 + 0,08 \cdot 1,6 = 0,771;$$

$$x_3^{(1)} = 0,95 + 0,1x_1^{(1)} + 0,05x_2^{(1)} + 0,1x_4^{(0)} - 0,15x_5^{(0)} = 0,95 + 0,1 \cdot 0,881 +$$
$$+ 0,05 \cdot 0,771 + 0,1 \cdot 1 - 0,15 \cdot 0,16 = 0,937;$$

$$x_4^{(1)} = 1 - 0,1x_2^{(1)} + 0,1x_3^{(1)} + 0,5x_5^{(0)} = 1,817;$$

$$x_5^{(1)} = 1,6 + 0,05x_1^{(1)} + 0,1x_2^{(1)} + 0,05x_3^{(1)} + 0,1x_4^{(1)} = 1,948$$

Next they approach In **the table** cited

This on the ground **Theorem** 's condition o ‘ rinly that it was for simple to iteration relatively Seidel iteration faster is approaching .

Table

k	$x_1^{(k)}$	$x_2^{(k)}$	$x_3^{(k)}$	$x_4^{(k)}$	$x_5^{(k)}$
0	0,6	0,44	0,95	1	1,6
1	0,881	0,771	0,937	1,817	1,948
2	0,973	0,961	0,985	1,974	1,992
3	0,995	0,995	0,999	1,996	1,999
4	0,9995	0,9991	0,9997	1,9995	1,9998
5	0,99992	0,99989	0,99997	1,99991	1,99997
6	0,99999	0,99998	0,99999	1,99999	1,99999

Conclusion

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Article Seidel method in the field of linear algebra important iterative method as instead in detail illuminates . Simple iteration from the method different Goodbye , Seidel . method consecutively approach scheme and his/her approach conditions theorem and proofs through clear Matrices typical by the modulus of numbers suddenly small to be of the method successful approach for main condition that is emphasized . Practical example through 5 rooms of the method in accuracy solution in finding efficiency shown . Seidel method , relevant conditions when done , simple to iteration relatively faster and reliable solution to give with separated it stands , this and him/her linear systems in solution important to the tool turns . Article researchers and students for this of the method theoretical and practical aspects in understanding valuable source is considered .

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