

OPTIMAL LOSS METHOD.

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Annotation: This article discusses the theoretical foundations and practical application of the optimal loss method. The essence of the method is that it implies decision-making aimed at minimizing losses while managing the activity of an object or system. The article analyzes the optimal loss criteria, the methods of their calculation and their application to real economic and technical questions. It also outlines strategies for determining optimal losses based on statistical and probability methods. The results of the study serve to increase the effectiveness of the optimal loss method and improve decision-making processes.

Keywords: optimal loss, loss function, decision theory, probability model, cost-effectiveness, management strategy, statistical estimation, minimization method.

Log in

Most of the modern scientific and practical problems are closely related to optimal decision-making. Any decision necessarily comes with a certain degree of risk and the possibility of loss. Therefore, one of the most important tasks in decision-making is to minimize losses – that is, choose the optimal path.

The optimal loss method is a technique that serves to find the optimal solution by identifying the loss function in complex systems, processes, or management problems and reducing it to a minimum value. This method is widely used in economics, management, engineering, artificial intelligence, medicine and many other fields.

The method is theoretically based on decision theory and probability analysis and makes it possible to identify losses in real conditions, model them, as well as to develop optimal solutions. This article analyzes the concept of the optimal loss method, its main types and the possibilities of applying them to practical issues.

The first steps of this method are similar to the Gaussian method. Assuming $a \neq 0$ that the lead element is,

[illegible]

The first equation of the system

$$x_1 + b_{12}^1 x_2 + \dots + b_{1n}^1 x_n = b_{1, n+1}^1 \quad (2)$$

to the appearance of a new species. Then we lose only the second equation of

(1): x_1

$$a_{22}^{(1)}x_2 + \dots + a_{2n}^{(1)}x_n = c_{2,n+1}^{(1)}$$

Now $a_{\gamma\gamma}^{(1)} \neq 0$, supposing that, let's put this equation into (3) as follows:

$$x_2 + b_{23}^{(2)}x_3 + \dots + b_{2n}^{(2)}x_n = b_{2,n+1}^2 \quad (3)$$

With this equation, we get rid of equation (2_x). As a result

$$x_1 + c_{13}^{(2)} x_3 + \dots + c_{1n}^{(2)} x_n = c_{1,n+1}^{(2)}$$

$$x_2 + c_{23}^{(2)} x_3 + \dots + c_{2n}^{(2)} x_n = c_{2,n+1}^{(2)}$$

is formed, here

$$c_{ij}^{(2)} = b_{1i}^{(1)} - b_{2i}^{(1)}b_{2i}^{(2)}, c_{2i}^{(2)} = b_{2i}^{(2)} \quad (j \geq 3).$$

k Suppose substitutions on the previous equations

As a result of the implementation of (1)system, the following equally strong system-

To be cited:

$$\left\{ \begin{array}{l} x_1 + c_{1,k+1}^{(k)}x_{k+1} + ... + c_{1n}^{(k)}x_n = c_{1,n+1}^{(k)} \\ \\ x_k + c_{k,k+1}^{(k)}x_{k+1} + ... + c_{kn}^{(k)}x_n = c_{k,k+1}^{(k)} \\ \\ a_{k+1,l}x_1 + ... + a_{k+1,k+1}x_{k+1} + ... + a_{k+1,n}x_n = a_{k+1,n+1} \\ \\ a_{n1}x_1 + ... + a_{n,k+1}x_{k+1} + ... + a_{nn}x_n = a_{n,n+1} \end{array} \right. \quad (4)$$

We multiply k the previous equation of this system $a_{k+1,k}, a_{k+1,2}, \dots, a_{k+1,k}$ by the corresponding ones, $(k+1)$ subtract the results from the equation and divide the resulting equation x_{k+1} by the coefficient before the infinite. The resulting $(k+1)$ equation would look like this:

$$x_{k+1} + c_{k+1,k+2}^{(k)} x_{k+2} + \dots + c_{k+1,n}^{(k)} x_n = c_{k+1,n+1}^{(k)}$$

Now, with the help of this equation, if we lose the previous equation of the system (4), k x_{k+1} then we have again the system of form (4), only k the form of $(k+1)$. . . At the same time, if

$$a_{k+1,k+1} - \sum_{r=1}^k c_{r,k+1}^{(k)} a_{k+1,r} \neq 0$$

In this case, we get the following formulas:

$$C_{k+1,p}^{(k+1)} = \frac{a_{k+1,p} - \sum_{r=1}^k a_{k+1,r} c_{r,p}^{(k)}}{a_{k+1,k+1} - \sum_{r=1}^k a_{k+1,r} c_{r,k+1}^{(k)}},$$

$$c_{ip}^{(k+1)} = c_{ip}^{(k)} - c_{i,k+1}^{(k)} c_{k+1,p}^{(k+1)}$$

$$(i=1,2,...,k; p=k+2,k+3,...,n+1).$$

After the n -step of substitutions n is also determined, the following formulas are generated for the solution of (1) system mapping:

$$x_i = c_{i,n+1}^{(n)} \quad (i = 1, 2, \dots, n).$$

Here, too, the control of the computational process is similar to the Gaussian method. All leading even in the optimal method of loss

It is necessary that the elements must be different from zero. If this fact is not known in advance, then it is more expedient to change the calculation scheme and eliminate the unknowns by selecting the main elements line by line. To do this, after $(k+1)$ eliminating unknowns x_1, x_2, \dots, x_k from the equation,

$$a_{k+1,k+1} - \sum_{s=1}^k a_{k+1,s} c_{sp}^{(k+1)} \quad (p > k+1)$$

is the largest element by modulus, then it is necessary to redefine the variables: $x_{k+1} = x_p$ and $x_p = x_{k+1}$, and then continue to eliminate the unknowns according to the optimal loss rule.

The advantage of the optimal loss method is that

Although the number of arithmetic operations required to solve an ordinal system is the same as in the Gaussian method, this method allows efficient use of computer memory, i.e., doubling the order of the system. (4) It can be seen from the system that once the step of optimal loss is k completed, the last equation of a given system $(n-k)$ remains unchanged. Taking this into account, we will introduce one line before each step, without fully entering all the elements of the matrix into the memory. In this case $(k+1)$, one memory cell will be enough to perform the step $f(k) = k(n-k+1) + n+1$, which

$$\begin{bmatrix} c_{1,k+1}^{(k)} & \dots & c_{1,n+1}^{(k)} \\ \dots & \dots & \dots \\ c_{k,k+1}^{(k)} & \dots & c_{k,n+1}^{(k)} \end{bmatrix}$$

It serves to accommodate the matrix and the coefficients of the equation in (4). $(k+1)$ Now $f(k)$ that we have found the maximum of, we n make sure that the area with the cell is sufficient to solve the ordered system $\frac{(n+1)(n+5)}{4}$. For example,

solving a system of 122-order equations on a computer with 4095 operating memory without using peripherals or calculating the determinant of an optional matrix of this order

It is possible.

As an example

$$\begin{cases} 2x_1 + 4,2x_2 + 1,6x_3 - 3x_4 = 3,2 \\ -0,4x_1 + 3x_2 - 2,4x_3 = -1,6 \\ 1,6x_1 - 0,8x_2 + x_3 - x_4 = -1 \\ x_1 - 2x_2 - x_3 + 1,5x_4 = 0 \end{cases}$$

Let's solve the system with the optimal method of elimination. First Equation

$$x_1 + 2,1x_2 + 0,8x_3 - 1,5x_4 = 16 \quad (5)$$

and multiply this by $-0,4$ and subtract from the second equation of the system:

$$3,84x_2 - 2,08x_3 - 0,60x_4 = -0,96$$

We divide this by 3.84 and form the required equation:

$$x_2 - 0,54167x_3 - 0,15625x_4 = -0,2500 \quad (6)$$

Where's (2.13) and x_2 Yo'qotsak,

$$x_1 + 1,93750x_2 - 1,17182x_4 = 2,12501 \quad (7)$$

If we divide (7) by 1.6 and (6) by -0.8 from the third equation of the system, and divide the resulting equation x_3 by the coefficient before it,

$$x_3 - 0,29611x_4 = 1,81556 \quad (8)$$

origin.

Using this equation, we eliminate (6) and (7) dai x_3 ,

$$\begin{cases} x_1 - 0,59811x_4 = -1,39322 \\ x_2 - 0,31664x_4 = 0,73343 \end{cases} \quad (9)$$

is formed.

Now we lose the x_1, x_2, x_3 fourth equation of the system using equations (8) -

(9): $1,11872x_4 = 1,67564$. From this and from (5)-(9) we find the unknowns in sequence:

$$x_4 = 4,00065; x_3 = 3,0019; x_2 = 1,99999; x_1 = 0,99922$$

Calculating the determinant.

Both the Gaussian method and the optimal loss method can be used to calculate the determinant. Following

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix}$$

Be required to find the determinant of the matrix. For this, homogeneous, linear

$$A\bar{x} = \bar{0} \quad (10)$$

To solve the system we apply the Gaussian method. The resulting A matrix

$$B = \begin{pmatrix} 1 & b_{12}^{(1)} & \dots & b_{1n}^{(1)} \\ 0 & 1 & \dots & b_{2n}^{(2)} \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 \end{pmatrix}$$

The triangle is replaced by a matrix, and the system (10) is equivalent to it

$$B\bar{x} = \bar{0}$$

system.

If we pay attention, the B elements of the matrix A are formed as a result of the following two elementary substitutions from A_1, A_2, \dots, A_{n-1} the matrix and the subsequent auxiliary matrices:

1) the division into hypothetical $a_{11}, a_{22}^{(1)}, \dots, a_{nn}^{(n-1)}$ leading elements as different from "zero";

2) separating rows from rows of matrix and helper A_1, A_2, \dots, A_{n-1} rows that are proportional to the leading rows, respectively.

As a result of the first substitution, the determinant of the matrix is also divided into a corresponding leading element, while the second substitution leaves the determinant unchanged. That's why

$$1 = \det B = \frac{\det A}{a_{11}, a_{22}^{(1)}, \dots, a_{nn}^{(n-1)}}$$

From this place ESA

$$\det A = a_{11}, a_{22}^{(1)}, \dots, a_{nn}^{(n-1)} \quad (11)$$

So the determinant is equal to the multiplication of the leading elements in the Gaussian compact scheme.

The matrix determinant can also be calculated using the optimal loss method. Here, too, the determinant leads all

$$a_k = a_{k+1,k+2} - \sum_{r=1}^k a_{k+1,r} c_{r,k+1}^{(k)}$$

is equal to the multiplicity of elements:

$$\det A = \bigcap_{k=1}^n a_k \quad (12)$$

If any of the leading elements is zero, then the scheme for selecting the head element by line should be used. But in this case a_k , it $(-1)^{l_k+1}$ would be necessary to multiply the elements by in order to preserve the determinant's gesture. The number here l_k refers to the number of unknowns lost in the step if k all unknowns that were not lost in the previous step are $1, 2, \dots, n-k$ numbered in $k+1$ successive sequences from left to right. But when the calculation is performed with the formulas (11) or (12) as usual, the multiplication of the former factors $\det A$ for $i < n$ something can be equal to or multiplied by the machine zero, unless i it is small (large).

To get rid of such a defect, it is necessary to calculate according to the formula (11) $\det A$ as follows:

$$\det A = (q \bigcap_j (-1)^{l_j+1} a_j) (r \bigcap_k (-1)^{l_k+1} a_k)$$

Here it is q close to the largest possible number in the exposition, close r to the smallest number, and at the same time $q \cdot r = 1$; a_j Among the leading elements, the ones that are smaller in modulus are the a_k remaining leading elements.

Now let's look at a case study of this method

Example: Solve the following system of equations using the optimal loss method.

$$\begin{cases} 2x + y - z = 8 \\ -3x - y + 2z = -11 \\ -2x + y + 2z = -3 \end{cases}$$

1. Let's pick a basic equation and leave it accessible to others to work with.

$2x + y - z = 8$ That's what we get.

2. From the other equations, we get x by using Equation 1.

Equation 2: $-3x - y + 2z = -11$

We multiply Equation 1 by 1.5:

$$1.5 \cdot (2x + y - z) = 3x + 1.5y - 1.5z = 12$$

Now we add Equation 2 and this result:

$$(-3x - y + 2z) + (3x + 1.5y - 1.5z) = -11 + 12$$

Result: $y + z = 2$ (new Equation 2)

Now we add Equation 3 and Equation 1:

$$(-2x + y + 2z) + (2x + y - z) = -3 + 8$$

Result: $2y + z = 5$ (new Equation 3)

3. We work with the 2 new equations in the second step

$$\begin{cases} y + z = 2 \\ 2y + z = 5 \end{cases}$$

We multiply Equation 1 by -1 and add to Equation 2:

$$(2y + z) - (y + z) = 5 - 2 \Rightarrow y = 3$$

Now we've found y. Let's put it in the first equation:

$$y + z = 2 \Rightarrow 3 + z = 2 \Rightarrow z = -1$$

4. X not even topamiz

We use Equation 1:

$$2x + y - z = 8 \Rightarrow 2x + 3 - (-1) = 8 \Rightarrow 2x = 4 \Rightarrow x = 2$$

Final solution: $x=2, y=3, z=-1$

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