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EQUATIONS SOLUTION HIGH ORDERLY DOG FOOD M E TODS . CHEBYSHEV'S METHOD.

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Annotation : This in the article **Chebyshev method** high orderly iterative methods explained.Given function roots in calculation Taylor formula using reverse function iterative solutions find process learns . In the article of the method main principles , Taylor formula and high orderly iterative of processes work principle in detail illuminated .

Key words : Chebyshev method , high orderly iterations , Taylor formula , inverse function , function root , iterative process , mathematics methods , precision , iterative methods .

Introduction . PL Chebyshev in 1933 given $f(x)$ to the function reverse was $g(y)$ function Taylor formula using to describe road with high orderly iteration build method offer It is assumed . let's do it , $f(x)=0$ of the equation $x=\xi$ root $[a,b]$ in between let it lie down and $f(x)$ function and his/her enough high orderly derivatives continuous Let it be . From now on outside this of the interval all at points $f'(x)\neq 0$ Let it be . Then $f'(x)$ this in between own gesture keeps and $f(x)$ monotonous function is , $x=g(y)$ reverse to the function has will be . Reverse function $g(y)$ $f(x)$ of change field

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$[c,d]$ identified in is , $f(x)$ how much continuous to derivatives has if so , that's it continuous to derivatives has will be . Reverse function to the definition according to

$$x \equiv g(f(x)) \quad (x \in [a,b]),\; y \equiv f(g(y)) \quad (y \in [c,d]). \; (1)$$

So,

$$\xi = g(0). \; (2)$$

If $y \in [c,d]$ If , then from Taylor's formula

$$\xi = g(0) = g(\nu - y) = g(y) + \sum_{k=1}^{p-1} (-1)^k \frac{g^{(k)}(y)}{k!} y^{(k)} + (-1)^p \frac{g^{(p)}(\eta)}{p!} y^p \quad (3)$$

this on the ground η number 0 and y between lies . Or y instead of $x = x_n$ what putting and $g(y) = x$ what in mind holding ,

$$\xi = x + \sum_{k=1}^{p-1} (-1)^k \frac{g^{(k)}(y)}{k!} f^{(k)}(x) + (-1)^p \frac{g^{(p)}(\eta)}{p!} f^p(x) \quad (4)$$

We generate . If

$$\varphi_p(x) = x + \sum_{k=1}^{p-1} (-1)^k \frac{g^{(k)}(f(x))}{k!} f^k(x)$$

If we define it as , then

$$x = \varphi_p(x) \; (5)$$

equation for $x = \xi$ solution will be , because

$$\varphi_p(\xi) = \xi + \sum_{k=1}^{p-1} (-1)^k \frac{g^{(k)}(f(\xi))}{k!} f^k(\xi) = \xi$$

From this

$$\varphi_p^j(\xi) = 0,\; j = \overline{1, p-1}$$

because of

$$x_{n+1} = \varphi_p(x_n) \quad (n = 0,1,2,\ldots;\, x_0 \in [a,b]) \, (6)$$

the iterative process is of p-order. If x_0 ξ to close If , then (6) with defined $\{ x_n \}$ sequence ξ to It is approaching . Indeed , $\varphi_p'(\xi) = 0$ it was for ξ of so surroundings It is found there. $|\varphi_p'(x)| \le q < 1$ It will be . and x_0 ξ to enough close if $\{ x_n \}$ iterative sequence

It comes close.

Now $\varphi_p(x)$ of $f(x)$ and We find its explicit expression through its derivatives. To do this, we take successive derivatives from (1):

$$\begin{cases} g'(f(x))f'(x)=1, \\ g''(f(x))f'^2(x)+g'(f(x))f''(x)=0, \\ g'''(f(x))f'^3(x)+3g''(f(x))f'(x)f''(x)+g'(f(x))f'''(x)=0, \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \end{cases} \quad (7)$$

From here we go one by one. $g'(f(x))$, $g''(f(x))$, ..., $g^{(p-1)}(f(x))$ those and this with together $\varphi_p(x)$ what We define . (6) We make the iteration process explicit for several specific values of p. $p=2$ when

$$\varphi_2(x)=x-\frac{f(x)}{f'(x)} \text{ and } x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)} \quad (8)$$

We will see later that this process is similar to the Newtonian process.

overlaps. $p=3$ When (5) and (7)

$$\varphi_3(x)=x-\frac{f(x)}{f'(x)}-\frac{f''(x)f^2(x)}{2[f'(x)]^3}$$

and

$$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}-\frac{f''(x_n)f^2(x_n)}{2[f'(x_n)]^3} \quad (9)$$

comes from. $p=4$ for

$$\varphi_4(x)=x_n-\frac{f(x)}{f'(x)}-\frac{f''(x)f^2(x)}{2[f'(x)]^3}-\frac{f^3(x)}{12}\cdot\frac{3f''^2(x)-f'(x)f'''(x)}{[f'(x)]^5}$$

and

$$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}-\frac{f''(x_n)f^2(x_n)}{2[f'(x_n)]^3}-\frac{f^3(x_n)}{12}\cdot\frac{3f''^2(x_n)-f'(x_n)f'''(x_n)}{[f'(x_n)]^5} \quad (10)$$

We generate . These iterative processes will be iterations of order 2, 3 , and 4, respectively.

Now $\varepsilon_n=\xi-x_n$ of mistake to zero We estimate the speed of aspiration . This for (4) in equality $x=x_n$, taking (6) as Considering this, we obtain the following:

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$$\xi - x_{n+1} = \frac{(-1)^p g^{(p)}(f(x))}{p!} f^p(x_n) \,(11)$$

this on the ground x ξ with , x_n between lies , $f(\xi)=0$ that was for

$$f(x_n) = -[f(\xi) - (fx_n)] = -(\xi - x_n) f'\overline{(x)} \qquad (12)$$

($\overline{(x)}$ also ξ with x_n lies between). We substitute (12) into (11):

$$\varepsilon_{n+1} = \frac{g^p(f(x))}{p!}[f'\overline{(x)}]^p \varepsilon_n^p \,(13)$$

The following

$$q = \max_{x,\overline{x}\in[a,b]} \left| \frac{g^p(f(x))}{p!}[f'\overline{(x)}]^p \right|$$

by introducing the definition, from (13)

$$|\varepsilon_{n+1}| \le q|\varepsilon_n|^p \,(14)$$

We get an inequality. Applying this inequality sequentially, we obtain the following:

$$|\varepsilon_n| \le q^{1+p+\ldots+p^{n-1}} |\varepsilon_0| p^n = (q|\varepsilon_0|)^{\frac{p^{n-1}}{p-1}} \varepsilon_0^{\frac{p^n(p-2)+1}{p-1}}$$

If $|\varepsilon_0|<1$ and $q|\varepsilon_0| = \omega < 1$, then

$$|\varepsilon_n| \le \omega^{\frac{p^{n-1}}{p-1}} \qquad (15)$$

becomes , which means that the iteration (6) converges extremely quickly

shows . Private without $\omega \le 10^{-1}$ and $|\varepsilon_0|<1$ If , for iterations (8), (9) and (10) above , we have the following, respectively:

$p=2$ for

$$|\varepsilon_1| \le 10^{-1},\ |\varepsilon_2| \le 10^{-3},\ |\varepsilon_3| \le 10^{-7},\ |\varepsilon_4| \le 10^{-15};\ldots$$

$p=3$ for

$$|\varepsilon_1| \le 10^{-1},\ |\varepsilon_2| \le 10^{-4},\ |\varepsilon_3| \le 10^{-13},\ |\varepsilon_4| \le 10^{-40},\ldots$$

$p=4$ for

$$|\varepsilon_1| \le 10^{-1},\ |\varepsilon_2| \le 10^{-5},\ |\varepsilon_3| \le 10^{-18},\ |\varepsilon_4| \le 10^{-85},\ldots$$

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So , $\omega \leq 0{,}1$ when third iteration himself/herself to us necessary accuracy gives .

Conclusion : Chebyshev method high orderly iterative from methods one Taylor formula using functions effective analysis to do opportunity This gives method iterations through of equations solutions fast and clear to find help The solution is p - order accuracy and of mistake to zero aspiration of the process efficiency increases . So so , Chebyshev method mathematician and practical in the fields effective application possible .

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