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SYSTEMS OF LINEAR EQUATIONS AND SOLVING THEM USING CRAMER'S RULE

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Abstract: In this scientific article, the theoretical foundations and practical application of solving systems of linear equations using Cramer's rule are described. The method is based on determining the unknowns through determinants and is applicable only when the main determinant is nonzero. The algorithm steps are demonstrated on examples with two and three unknowns, and the general form of the formulas is presented.

Keywords: Cramer's rule, trapezoid, system of linear equations, algebraic method, determinant, constant term.

Аннотация: В данной научной статье раскрываются теоретические основы и практическое применение решения систем линейных уравнений методом Крамера. Метод основан на определении неизвестных с помощью определителей и применяется только в случаях, когда главный определитель не равен нулю. На примерах с двумя и тремя неизвестными показаны этапы алгоритма и приведена общая форма формул.

Ключевые слова: метод Крамера, трапеция, система линейных уравнений, алгебраический метод, определитель, свободный член.

Systems of linear equations arise in many fields of science, engineering, and economics. Solving such systems efficiently and accurately is an important skill in linear algebra. One of the classical methods for solving systems with a unique solution is **Cramer's Rule**, which is based on the concept of determinants.

Suppose we are given a system of m linear equations in n unknowns:

$$\begin{cases} a_{1,1}x_1 + a_{1,2}x_2 + a_{1,3}x_3 + \cdots + a_{1,n}x_n = b_1 \\ a_{2,1}x_1 + a_{2,2}x_2 + a_{2,3}x_3 + \cdots + a_{2,n}x_n = b_2 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ a_{m,1}x_1 + a_{m,2}x_2 + a_{m,3}x_3 + \cdots + a_{m,n}x_n = b_m \end{cases} \quad (1)$$

Here, the variables $x_1, x_2, x_3, \cdots, x_n$ are the unknowns. We assume that the equations are numbered as the first, second, and so on, up to the m −th equation. The coefficient $a_{1,1}a_{2,1}, \cdots, a_{m,n}$ represents the coefficient of the $a_{i,j}$ −th unknown x_j in the i-th equation, while b_i denotes the constant term (free term) of the j-th equation.

A solution of a linear equation is defined as an arithmetic vector $\alpha_1, \alpha_2, \cdots, \alpha_n \in R^n$ of real numbers that satisfies the linear equation. The coefficients in front of the unknowns can be written in the form of a matrix consisting of m rows and n columns.

$$A = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \cdots a_{1,n} \\ \cdots & \cdots & \cdots \ \cdots \ \cdots \\ a_{m,1} & a_{m,2} & a_{m,3} \cdots a_{m,n} \end{bmatrix} \quad (2)$$

This matrix is referred to as the coefficient matrix of the system of linear equations, while the following matrix $\widetilde{A}$ is known as the augmented matrix of the system:

$$\tilde{A} = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \cdots a_{1,n} \ b_1 \\ \cdots & \cdots & \cdots \ \cdots \ \cdots \ \ \cdots \\ a_{m,1} & a_{m,2} & a_{m,3} \cdots a_{m,n} b_m \end{bmatrix} (3)$$

If all the constant terms of the coefficient matrix are equal to zero, then the system is called a homogeneous system of linear equations.

If the number of rows m in the coefficient matrix is equal to the number of columns n , then the system is referred to as an n-th order system.

A system of linear equations that has at least one solution is called a consistent system.

For example, any homogeneous system is consistent, since assigning zero to all unknowns yields a solution to the system.

A system that has a unique solution is called a *determinate system*, while a system that has more than one solution is referred to as an *indeterminate system*.

A system that has at least one solution is called a consistent system, while a system that has no solution is called an inconsistent system.

(1) In a system, the following operations are called ***elementary transformations*** if performed:

1. Multiplying both sides of a given k-th equation by a nonzero scalar α;
2. Interchanging any two equations in the system;
3. Multiplying two equations of the system by nonzero scalars $a \neq 0$ *and* $b \neq 0$ respectively, and adding the results;
4. Removing an equation consisting entirely of zero coefficients and a zero constant term (if such an equation exists).

A system resulting from such transformations is said to have undergone elementary transformations**.**

Theorem: *A system obtained through elementary transformations is equivalent to the original system.*

A homogeneous system of linear equations with ***n*** unknowns and ***m*** equations has a non-trivial solution if ***m*** < ***n***.

If all the elements below the main diagonal of a square matrix are zeros, such a matrix is called an *upper triangular matrix***.**

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$$A = \begin{pmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,n} \\ 0 & a_{2,2} & \cdots & a_{2,n} \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & a_{m,n} \end{pmatrix}$$

The method of Cramer for solving systems of linear equations.

The Cramer's rule method for solving a system of linear equations is applicable when the number of equations in the system is equal to the number of unknowns.

$$\begin{cases} a_{1,1}x_1 + a_{1,2}x_2 + a_{1,3}x_3 + \cdots + a_{1,n}x_n = b_1 \\ a_{2,1}x_1 + a_{2,2}x_2 + a_{2,3}x_3 + \cdots + a_{2,n}x_n = b_2 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ a_{m,1}x_1 + a_{m,2}x_2 + a_{m,3}x_3 + \cdots + a_{m,n}x_n = b_m \end{cases}$$

Let us denote the determinant of the matrix formed by the coefficients of the system of equations by the letter D.

$$D = \begin{pmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,n} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m,1} & a_{m,2} & \cdots & a_{m,n} \end{pmatrix}$$

For the use of Cramer's rule, it is necessary and sufficient that the determinant value is non-zero. If the determinant is equal to zero, using Cramer's method is incorrect.

$$\begin{cases} a_{1,1}x_1 + a_{1,2}x_2 + a_{1,3}x_3 + \cdots + a_{1,n}x_n = b_1 \\ a_{2,1}x_1 + a_{2,2}x_2 + a_{2,3}x_3 + \cdots + a_{2,n}x_n = b_2 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ a_{m,1}x_1 + a_{m,2}x_2 + a_{m,3}x_3 + \cdots + a_{m,n}x_n = b_m \end{cases}$$

We are given a system in this form. We form the determinant from the coefficients of the unknowns as follows:

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$$D = \begin{pmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,n} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m,1} & a_{m,2} & \cdots & a_{m,n} \end{pmatrix}$$

We calculate the value of the determinant, and if it equals zero, we use the Gauss method instead of this method. If it is different from zero, then Cramer's method is definitely appropriate.

Since the determinant D is non-zero, the given system has a unique solution.

$$D \neq 0$$

$$D_1 = \begin{pmatrix} b_1 & a_{1,2} & \cdots & a_{1,n} \\ b_2 & a_{2,2} & \cdots & a_{2,n} \\ \cdots & \cdots & \cdots & \cdots \\ b_m & a_{m,2} & \cdots & a_{m,n} \end{pmatrix} \qquad D_2 = \begin{pmatrix} a_{1,1} & b_1 & \cdots & a_{1,n} \\ a_{2,1} & b_2 & \cdots & a_{2,n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m,1} & b_m & \cdots & a_{m,n} \end{pmatrix}$$

$$\cdots\cdots\cdots\cdots$$

$$D_n = \begin{pmatrix} a_{1,1} & a_{1,2} & \cdots & b_1 \\ a_{2,1} & a_{2,2} & \cdots & b_2 \\ \cdots & \cdots & \cdots & \cdots \\ a_{m,1} & a_{m,2} & \cdots & b_m \end{pmatrix}$$

$$x_1 = \frac{D_1}{D} \qquad x_2 = \frac{D_2}{D} \qquad \cdots\cdots\cdots\ x_n = \frac{D_n}{D}$$

For Answer: $(x_1, x_2, \cdots, x_n)$

The easiest way to teach this method is by looking at an example.

$$\begin{cases} 2x - y + z = 2 \\ 3x + 2y + 2z = -2 \\ x - 2y + z = 1 \end{cases}$$

$$D = \begin{pmatrix} 2 & -1 & 1 \\ 3 & 2 & 2 \\ 1 & -2 & 1 \end{pmatrix} = 2 \times 2 \times 1 + 1 \times 3 \times (-2) + (-1) \times 2 \times 1 - (1 \times 2 \times 1 + 2 \times 2 \times (-2) + 3 \times (-1) \times 1) = 4 - 6 - 2 - (2 - 8 - 3) = -4 + 9 = 5$$

$$D \neq 0$$

$$D_x = \begin{pmatrix} 2 & -1 & 1 \\ -2 & 2 & 2 \\ 1 & -2 & 1 \end{pmatrix} = 10$$

$$D_y = \begin{pmatrix} 2 & 2 & 1 \\ 3 & -2 & 2 \\ 1 & 1 & 1 \end{pmatrix} = -5$$

$$D_z = \begin{pmatrix} 2 & -1 & 2 \\ 3 & 2 & -2 \\ 1 & -2 & 1 \end{pmatrix} = -15$$

$$x_1 = \frac{10}{5} = 2 \quad x_2 = \frac{-5}{5} = -1 \quad x_3 = \frac{-15}{5} = -3$$

Answer:(2,-1,-3)

In conclusion, I would like to emphasize that for systems with a non-zero determinant, Cramer's rule is one of the simplest and most convenient methods to understand and apply.

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