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FIBONACCI NUMBERS AND THEIR SIGNIFICANCE IN VARIOUS FIELDS

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Abstract Fibonacci numbers play a significant role in various branches of mathematics. This paper explores the properties of the Fibonacci sequence, methods for investigating them, and their practical applications. Additionally, an algorithmic approach to checking Fibonacci numbers using programming is presented. The study highlights the importance of Fibonacci numbers in natural processes, algorithms, and economics.

Keywords: Fibonacci numbers, recursion, algorithms, verification methods, programming, Leonardo, iterative method, financial modeling.

1. Introduction

The Fibonacci sequence, introduced by Leonardo Fibonacci in 1202, is widely applied in natural phenomena, various fields of mathematics, and algorithmic processes. This sequence is defined by the recurrence relation:

$$F(n) = F(n-1) + F(n-2), \qquad F(0) = 0, F(1) = 1.$$

Fibonacci numbers have unique properties that make them fundamental in both theoretical and applied mathematical research.

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2. Methods for Verifying Fibonacci Numbers

Various methods exist for computing and verifying Fibonacci numbers:

2.1 Recursive Method

Using recursion, Fibonacci numbers can be computed as follows:

$$F(n) = F(n-1) + F(n-2)$$

However, this approach is inefficient for large values due to redundant calculations.

2.2 Iterative Method

An iterative approach calculates Fibonacci numbers using loops, improving computational efficiency by reducing redundant calculations.

2.3 Matrix Multiplication Method

A more efficient method involves matrix exponentiation:

$$\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n \times \begin{bmatrix} F(0) \\ F(1) \end{bmatrix} = \begin{bmatrix} F(n+1) \\ F(n) \end{bmatrix}$$

This method is particularly useful for handling large Fibonacci numbers.

2.4 Binet`s Formula

To find the nth Fibonacci number directly without recursion, we use Binet’s formula:

$$F(n) = \frac{P^n - \phi^n}{\sqrt{5}}$$

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Where $P = \frac{\sqrt{5}+1}{2}$ (Golden Ratio) and $\phi = \frac{1-\sqrt{5}}{2}$.

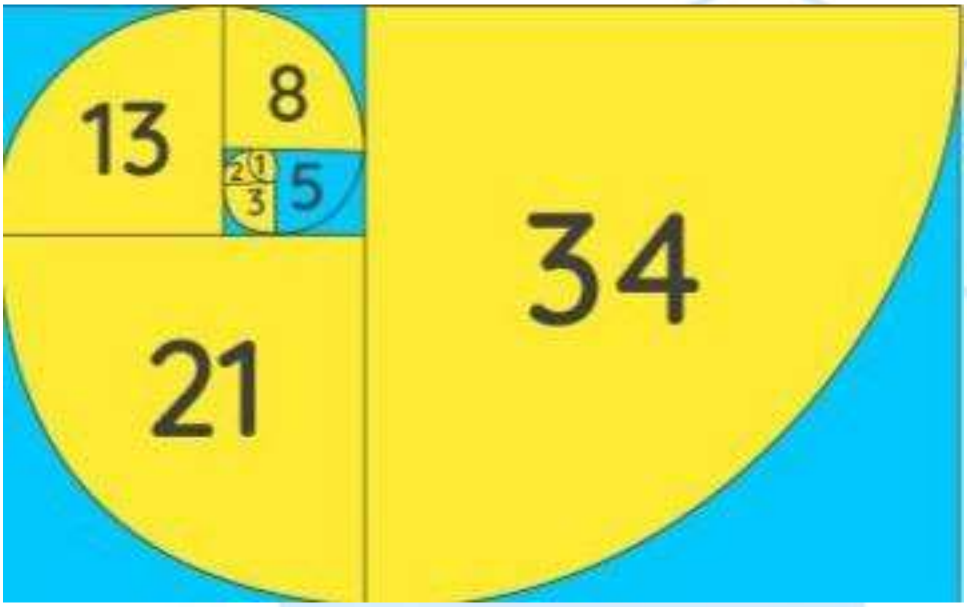
2.5 Fibonacci Spiral

The Fibonacci spiral is a geometric representation of the Fibonacci sequence, constructed using the following property of the sequence (which can be proven by mathematical induction):

$$F(1)^2 + F(2)^2 + \cdots + F(n)^2 = F(n)F(n+1)$$

We begin with two adjacent squares of side length 1. From the formula, we know that the length of the resulting rectangle will be $F(n + 1$Therefore, we add a square with a side length of $F(n + 1)$ attaching it to the longer edge of the existing rectangle. Now, the overall shape forms a new rectangle with a side length of $F(n + 2)$. Next, we add another square of side length $F(n + 2)$, and this process continues iteratively.

By drawing a smooth curve through the corners of these squares, we create the Fibonacci spiral.



2.6 Fibonacci Numbers and the Golden Ratio

The ratio of consecutive Fibonacci numbers approximates the Golden Ratio (1,618034). For instance:

$$\frac{21}{13} \approx 1{,}615$$

Using this ratio, Fibonacci numbers can be computed as:

$$F_n = \frac{\left((\phi)^n - (1-\phi)^n\right)}{\sqrt{5}}$$

Where $\phi = 1{,}618034$.

3. Applications of Fibonacci Numbers

3.1 Use in Computer Algorithms

Fibonacci numbers are essential in dynamic programming, fast search algorithms, and data compression techniques.

3.2 Financial Modeling

Fibonacci ratios $(61.8\%, 38.2\%, and\ 23.6\%)$ are extensively used in stock market analysis and technical analysis.

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4. Fibonacci Number Calculation Program

The following C# program calculates Fibonacci numbers using an iterative approach:

```
using System;
```

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```
class Program
{
    static void Main(string[] args)
    {
        Console.Write("Enter the Fibonacci sequence length: ");
        int n = int.Parse(Console.ReadLine());
        Fibonacci(n);
    }
    static void Fibonacci(int n)
    {
        int a = 0, b = 1, c;
        Console.Write(a + " " + b + " ");
        for (int i = 2; i < n; i++)
        {
            c = a + b;
            Console.Write(c + " ");
            a = b;
            b = c;
        }
```

```
    }
}
```

Examples

Example 1: Compute the 12th and 13th Fibonacci numbers:

The 9th and 10th terms in the sequence are 21 and 34.

$$F_{11} = 21 + 34 = 55$$

$$F_{12} = 34 + 55 = 89$$

$$F_{13} = 55 + 89 = 144$$

Example 2: Given $F_{14} = 377$, compute F_{15}.

Using the Golden Ratio:

$$F_{15} = F_{14} \times 1{,}618 \approx 377 \times 1{,}618 = 610$$

Answer: $F_{15} = 610$.

5. Conclusion

This paper has examined the properties, computation methods, and applications of Fibonacci numbers. The findings indicate that Fibonacci numbers are not only a mathematical curiosity but also play a vital role in solving real-world problems across multiple disciplines, including algorithms, nature, and finance

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