A HIROTA BILINEAR METHOD FOR A COMPLEX PERTURBED MODIfiED KORTEWEG-DE VRIES EQUATION

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Abstract. In this paper by using Hirota direct method, the one-soliton solution of perturbed modified Korteweg-de Vries equation mKdV are studied. We have shown the evolution of the one-soliton solutions.

Keywords: soliton solution, Schrodinger-Hirota equation, nonlinear equations, Hirota direct method.

1. Introduction

Meanwhile, a variety of powerful methods for seeking the explicit and exact solutions of nonlinear evolution equations have been proposed and developed. Among them are the inverse scattering method [1], Hirota's direct method, Backlund transformation method, Darboux transformation method, tanh-sech method [2–4], extended tanh method [5–7], sine-cosine method [8–10], homogeneous balance method [11, 12], Jacobi elliptic function method [13–16], *F*-expansion method [17–19], exp-function method [20, 21], trigonometric function series method [22], (Gў/ G)-expansion method [23, 24] and so on.

Among them, the Hirota method can solve not only integrable equations, but also non-integrable equations, the Hirota bilinear method is an important and direct method [25]. The advantage of the Hirota bilinear method is an algebraic rather than analytical method. And it has been successfully applied to solve a large number of soliton equations, for examples, KdV equation, mKdV equation, Sine-Gordon equation, nonlinear Schrödinger equation, etc. Some analytic dark two-soliton solutions of highorder nonlinear Schrödinger equation are obtained via the Hirota bilinear method in the inhomogeneous optical fiber, and some new phenomena are presented for the

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first time in [26]. Optical soliton propagation is investigated theoretically in the dispersion-decreasing fibers, and three solitons can split into four solitons or merge into two solitons in [27]. The dynamics of spatiotemporal solitons of a partially nonlocal Schrödinger equation are constructed via Darboux transformation method and Hirota method in [28]. Numerical solutions of the nonlinear Schrödinger equation studied in [29-31].

The main content of this work is to solve the complex mKdV equation using the improved Hirota bilinear method. We give the one-soliton solution.

2. Hirota bilinear method for a complex perturbed mKdV equation

Perturbed NLSE with Kerr law nonlinearity [28] with following form:

$$iq_t + q_{xx} + a\,|q|^2 q + i[g_1 q_{xxx} + g_2 |q|^2 q_x + g_3 q(|q|^2)_x] = 0 .$$

When $a = 0, g_1 = 1, g_2 = 0, g_3 = 1$ we have complex perturbed MKdV equation

$$q_t + q_{xxx} + 6|q|^2 q_x + 3q(|q|^2)_x = 0, \qquad (1)$$

where $q = q(x,t)$ is a complex-valued function of real variables *x* and *t*.

We take an improved Hirota bilinear method to Eq. (1) simultaneously by introducing the dependent variable transformations, namely

$$q(x,t) = \frac{g(x,t)}{f(x,t)}, \qquad (2)$$

where the $g = g(x,t)$ is complex-valued function, the $f = f(x,t)$ is real-valued function.

We substitute the transformation Eq. (2) into Eq. (1), and have a novel system

$$\left(\frac{g}{f}\right)_t + \left(\frac{g}{f}\right)_{xxx} + 6\left|\frac{g}{f}\right|^2 \left(\frac{g}{f}\right)_x + 3\left(\frac{g}{f}\right)\left(\left|\frac{g}{f}\right|^2\right)_x = 0 \qquad (3)$$

which is equivalent to

$$\frac{1}{f^2}(D_t + D_x^3)gf + \frac{1}{f^4}\left((9|g|^2 - 3D_x^2 ff)D_x gf - 3g^2 D_x \bar{g}f\right) = 0 \qquad (4)$$

Hence the Hirota bilinear forms of the integrable complex MKdV equation are given rise to as follows

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$$(D_t + D_x^3) g(x,t) f(x,t) = 0\,, \tag{5}$$

$$(9|g|^2 - 3D_x^2 ff) D_x gf - 3g^2 D_x \bar{g} f = 0\,. \tag{6}$$

In the above equations, the *Dt* and *Dx* are the standard Hirota's bilinear operators and they are defined as

$$D_x^m D_t^n g(x,t) \cdot f(x,t) = \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'}\right)^m \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'}\right)^n g(x,t) f(x',t')|_{x=x',t=t'} \tag{7}$$

Some soliton solutions can be obtained by solving the above set of bilinear Eqs. (5) and (6). In this section, we expand the unknown functions $g(x,t)$, $f(x,t)$ in terms of a small parameter ε

$$g = \varepsilon g_1 + \varepsilon^3 g_3 + \ldots, \quad g^* = \varepsilon g_1^* + \varepsilon^3 g_3^* + \ldots \tag{8}$$

$$f = 1 + \varepsilon^2 f_2 + \varepsilon^4 f_4 + \ldots, \tag{9}$$

where the g_1, g_3 are functions of x and t, the g_1^*, g_3^* are functions of x and t. Substituting the above expansions into Eqs. (5) and (6), and equating the coefficients of same powers of ε to zero, we obtain a set of equations for the unknown functions $g(x,t)$, $g^*(x,t)$ and $f(x,t)$. Some solutions of the soliton equations can be obtained by selecting the appropriate functions g_1, g_1^*, g_3, g_3^*, etc.

3. One soliton solution of Eq. (1)

In order to construct one-soliton solution of complex MKdV equation (1), we take the following expansions of the functions g, g^*, f and f^*:

$$g(x,t) = \varepsilon g_1, \quad g^*(x,t) = \varepsilon g_1^*, \tag{10}$$

$$f(x,t) = 1 + \varepsilon^2 f_2, \tag{11}$$

Substituting the above expansions of Eq. (11) into the bilinear Eqs. (5), (6) and (7), we take the procedure of an improved Hirota bilinear method for a complex MKdV equation. We can derive a series of linear partial differential equations of the unknown functions g_1, g_1^* and f_2 as follows

$$(D_t + D_x^3) g_1 = 0\,, \tag{12}$$

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in which, the following forms of $g_1(x,t)$, $g_1^*(x,t)$ and $f_2(x,t)$ are consistent with Eq. (12), which are given rise to as follows

$$g_1(x,t)=e^{\eta_1},\ g_1^*(x,t)=e^{\bar{\eta}_1}, \quad (13)$$

$$f_2(x,t)=e^{\eta+\bar{\eta}_1+A_1},\ e^{A_1}=\frac{1}{(k_1-\bar{k}_1)^2} \quad (14)$$

Where $\eta_1=k_1x-w_1t+\xi_1$,$\bar{\eta}_1=-\bar{k}_1x-\bar{w}_1t+\bar{\xi}_1$ and k_1, $\bar{k}_1$, A_1 are arbitrary complex constants. When we take Eqs. (13) and (14) into Eq. (12), then obtain $k_1^3=w_1$ and the function f_2 as follows

$$f_2=\frac{e^{\eta_1+\bar{\eta}_1}}{\left(k_1-\bar{k}_1\right)^2}=\frac{e^{\left(k_1-\bar{k}_1\right)x-(k_1^3+\bar{k}_1^3)t}}{\left(k_1-\bar{k}_1\right)} \quad (15)$$

Substituting the above expressions into the relevant expressions given in Eqs. (9), (10) and rewriting them suitably, we derive the one - soliton solution of complex MKdV equation (1)

$$q(x,t)=\frac{2ie^{kx-\omega t}}{1+e^{2kx-2\omega t}},\ e^{A_1}=\frac{1}{\left(k_1-\bar{k}_1\right)^2}. \quad (16)$$

According to the bilinear form of parity transformed complex conjugate equation, we can obtain the parity transformed complex conjugate field in the form

$$q^*(x,t)=\frac{2ie^{\bar{k}x-\bar{\omega}t}}{1+e^{2\bar{k}x-2\bar{\omega}t}}\ . \quad (17)$$

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