ISSN 2181-3187

BITTA DINAMIK SISTEMANING TAHLILI HAQIDA

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Annotatsiya: Maqolada kvadratik stoxastik operatorlar sinfi va tavsifi keltirilgan. Dinamik sistemalar orqali ifodalanuvchi jarayonning tasniflari, ya'ni dinamik sistemalarni to'liq klassifikatsiyasi keltirilgan. O'rganilgan uzluksiz vaqtli kvadratik stoxastik operator bilan diskret vaqtli kvadratik stoxastik operatorlarning qiyosiy tahlili keltirilib, sonli usullar natijalari bilan ustma-ust tushishi ko'rsatilgan.

Kalit so'zlar: kvadratik stoxastik operator, klassifikatsiya, ehtimollik taqsimoti, sonli yechimlar.

ABOUT THE ANALYSIS OF A DYNAMIC SYSTEM

Abstract: The article presents a class and description of quadratic stochastic operators. Classification of processes represented by dynamic systems, in a complete classification of dynamic systems are given. A comparative analysis of discrete time quadratic stochastic operators with the studied continuous time quadratic stochastic operator is presented, and it is shown that the theoretical results overlap with the results of numerical methods.

Keywords: quadratic stochastic operator, classification, probability distribution, numerical solutions.

ISSN 2181-3187

Biz o'rganayotgan ishda kvadratik stoxastik operatorning ta'rifi quyidagicha keltirilgan:

$$E = \{1, 2, \ldots, n\} \ \text{bo'lsin.}$$

$$S^{n-1} = \left\{ x = \{x_1, x_2, \ldots, x_n\} \in R^n : x_i \geq 0, \sum_{i=1}^{n} x_i = 1 \right\}$$

to'plam $(n-1)$ o'lchovli simpleks deb ataladi. Har bir $x \in S^{n-1}$ element E da ehtimolli kattalik bo'lib, uni n ta elementdan tashkil topgan biologik sistema holati kabi izohlash mumkin. Kvadratik stoxastik operator $V: S^{n-1} \to S^{n-1}$

$$V: x'_k = \sum_{i,j=1}^{n} \rho_{ij,k} x_i x_j, \qquad (1)$$

bu yerda $\rho_{ij,k}$ irsiylik darajasi bo'lib, u quyidagi shartlarni qanoatlantiradi:

$$\rho_{ij,k} \geq 0, \qquad \rho_{ij,k} = \rho_{ji,k}, \qquad \sum_{k=1}^{n} \rho_{ij,k} = 1.$$

Matematik genetikada ushbu operator V populyatsiyaning evolutsion operatori yoki kvadratik stoxastik operatorlar sinfiga kiruvchi operator hisoblanadi. Populyatsiya ko'paytirish amaliga nisbatan yopiq bo'lgan organizmlar birlashmasi sifatida tavsiflanadi [1-15]. Populyatsiyada $F_1, F_2, \ldots$ nasllar ketma-ketligi farqlanadi. Turli nasllar vakillari o'rtasida kesishish sodir bo'lmaydi deb taxmin qilamiz. Populyatsiya tarkibiga kiradigan har bir vakil 1,2,…,n turlardan biriga tegishli. Populyatsiya holati $x = (x_1, \ldots, x_n) \in S^{n-1}$.

Ushbu maqolada kvadratik stoxastik operatorning (1) uzluksiz analogining muayyan holatini, ya'ni chiziqli bo'lmagan oddiy differensial tenglamalar sistemasini o'rganamiz. Bunda $n = 4$ va $\rho_{ij,k}$ ba'zi qiymatlarida o'rganilayotgan uzluksiz vaqtli dinamik sistema quyidagi ko'rinishga ega bo'ladi:

$$\begin{cases} \dot{x_1} = x_1 y_1 - x_1, & (1) \\ \dot{x_2} = 0, & (2) \\ \dot{y_1} = x_1 + x_2 y_1 - y_1, & (3) \\ \dot{y_2} = x_2 y_2 - y_2. & (4) \end{cases}$$

(2) tenglamadan $x_2 = c_2 = const$. Buni (4) tenglamaga etib qo'yamiz:

$$\dot{y_2} = c_2 y_2 - y_2 = (c_2 - 1) y_2.$$

1-hol. $c_2 - 1 = 0 \Rightarrow x_2 = 1 \Rightarrow x_1 + x_2 = 1 \Rightarrow x_1 = 0 \Rightarrow$ (3) tenglamadan

$$\dot{y_1} = x_1 + x_2 y_1 - y_1 = 0 + y_1 - y_1 = 0 \Rightarrow y_1 = c_3 = const \quad y_2 = 1 - c_3$$

$$\begin{cases} x_1 = 0, \\ x_2 = 1, \\ y_1 = c_3, \\ y_2 = 1 - c_3. \end{cases}$$

2-hol. $y_2 = 0 \Rightarrow y_1 + y_2 = 1 \Rightarrow y_1 = 1$ (2) tenglamadan $x_2 = c_2 = const$. $y_1 = 1$ ni (1) tenglamaga qo'ysak $\dot{x_1} = 0 \Rightarrow x_1 = c_1 = const$. (3) tenglamadan $0 = c_1 + c_2 - 1$. Demak

$$\begin{cases} x_1 = c_1, \\ x_2 = 1 - c_1, \\ y_1 = 1, \\ y_2 = 0. \end{cases}$$

3-hol. $x_2 = c_2\ (c_1 - 1) y_2 \neq 0$

$$\dot{y_2} = x_2 y_2 - y_2 = c_2 y_2 - y_2 = (c_2 - 1) y_2$$

$$\frac{dy_2}{y_2} = (c_2 - 1) dt \Rightarrow ln y_2 = (c_2 - 1) t + ln c_4 \quad \Rightarrow$$

$$y_2 = c_4 e^{(c_2 - 1)t}.$$

$x_2 = c_2$ bo'lsa, $x_1 + x_2 = 1$ dan $x_1 = 1 - c_2$ bo'ladi (1) va (3) tenglamalar quyidagi ko'rinishga keladi: $\dot{x_1} = 0 \Rightarrow$

$$\begin{cases} 0 = (1-c_2)y_1 - (1-c_2) & (5) \\ \dot{y_2} = c_2y_2 - y_2 + 1 - c_2 = (1-c_2)(1-y_1) & (6) \\ x_2 = c_2 & (7) \\ y_2 = c_4e^{(c_2-1)t} & (8) \end{cases}$$

(5) tenglamadan quyidagini olamiz:

$(1-c_2)(y_1-1) = 0.$

Bu tenglama ikki holda qaraladi. 1-hol.

$c_2 = 1 \Rightarrow x_1 = 0\,, x_2 = 1$ (8) tenglamadan $y_2 = c_4$ bo'ladi. Demak

$$\begin{cases} x_1 = 0, \\ x_2 = 1, \\ y_1 = 1 - c_4, \\ y_2 = c_4. \end{cases}$$

2-hol.

$y_1 = 1 \Rightarrow y_1 + y_2 = 1$ dan, $c_4 = 0$ deb olinadi. Demak,

$$\begin{cases} x_1 = 1 - c_2, \\ x_2 = c_2, \\ y_1 = 1, \\ y_2 = 0. \end{cases}$$

Endi sistemaning qo'zg'almas nuqtalarini topamiz.

$$\begin{cases} x_1y_1 - x_1 = 0 & (9) \\ 0 = 0 & (10) \\ x_1 + x_2y_1 - y_1 = 0 & (11) \\ x_2y_2 - y_2 = 0 & (12) \end{cases}$$

(9) tenglamadan: $x_1y_1 - x_1 = 0$, $x_1(y_1 - 1) = 0.$

1-hol. $x_1 = 0 \Rightarrow x_2 = 1, y_1 \neq 1$ (11) tenglamadan $y_1 = c_3 = const \Rightarrow$ (12) tenglamadan

$$y_2 = 1 - c_3 \;\Rightarrow$$

ISSN 2181-3187

$$\begin{cases} x_1 = 0, \\ x_2 = 1, \\ y_1 = c_3, \\ y_2 = 1 - c_3. \end{cases} \Rightarrow$$

$$M_1^{13}(0,1; c_3, 1 - c_3).$$

2-hol. $y_1 = 1 \Rightarrow y_2 = 0$, $x_1 \neq 1$ (11) tenglamadan $x_1 + x_2 - 1 = 0$ ga ega bo'lamiz. (9) tenglamadan $x_1 = c_1$ desak, $x_2 = 1 - c_1$ bo'ladi.

$$\begin{cases} x_1 = c_1 \\ x_2 = 1 - c_1 \\ y_1 = 1 \\ y_2 = 0 \end{cases} \Rightarrow$$

$$M_2^{13}(c_1, 1 - c_1; 1,0).$$

3-hol. $x_1 = 0$, $y_1 = 1 \Rightarrow x_2 = 1, y_2 = 0$ $M_3^{13}(0,1; 1,0)$.

$M_1^{13}(0,1; c_3, 1 - c_3)$ nuqta atrofida chiziqlashtiramiz:

$$\frac{\partial f_1}{\partial x_1} = y_1 - 1\big|M_1^{13} = c_3 - 1, \quad \frac{\partial f_1}{\partial x_2} = 0, \quad \frac{\partial f_1}{\partial y_1} = x_1\big|M_1^{13} = 0, \quad \frac{\partial f_1}{\partial y_2} = 0$$

$$\frac{\partial f_2}{\partial x_1} = \frac{\partial f_2}{\partial x_2} = \frac{\partial f_2}{\partial y_1} = \frac{\partial f_2}{\partial y_2} = 0$$

$$\frac{\partial f_3}{\partial x_1} = 1, \quad \frac{\partial f_3}{\partial x_2} = y_1\big|M_1^{13} = c_3, \quad \frac{\partial f_3}{\partial y_1} = x_2 - 1\big|M_1^{13} = 0, \quad \frac{\partial f_3}{\partial y_2} = 0$$

$$\frac{\partial f_4}{\partial x_1} = 0, \quad \frac{\partial f_4}{\partial x_2} = y_2\big|M_1^{13} = 1 - c_3, \quad \frac{\partial f_4}{\partial y_1} = 0, \quad \frac{\partial f_4}{\partial y_2} = x_2 - 1\big|M_1^{13} = 0$$

$$A = \begin{pmatrix} c_3 - 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & c_3 & 0 & 0 \\ 0 & 1 - c_3 & 0 & 0 \end{pmatrix}$$

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$$det|A-\lambda E| = \begin{vmatrix} c_3-1-\lambda & 0 & 0 & 0 \\ 0 & -\lambda & 0 & 0 \\ 1 & c_3 & -\lambda & 0 \\ 0 & 1-c_3 & 0 & -\lambda \end{vmatrix} = \lambda^3(\lambda+1-c_3)=0.$$

$\lambda_{1,2,3}=0,\ \ \lambda_4=c_3-1\quad \lambda_1=0$ xos soni 3 karrali ildiz.

Demak bu sistemaning noturg'unligini aniqlab bo'lmaydi.

Agar $c_3=0$ bo'lsa ham $s_1=2\delta-\delta_1, k_1\neq s_1$bo'ladi. Ushbu Sistema noturg'un [16-25]. Agar $c_3=1$ bo'lsa , $M_2^{13,1}(0{,}1;1{,}0)$ bo'ladi. $rankA=1$, $s_i=3$ va $s_1\neq k_1$ bo'ladi. Sistema noturg'un.

M_1^{13} nuqta atrofida sistemani ko'rinishi quyidagicha bo'ladi.

$$\begin{cases} \dot{x_1}=(c_3-1)x_1, \\ \dot{x_2}=0, \\ \dot{y_1}=x_1+c_3(x_2-1), \\ \dot{y_2}=(1-c_3)(x_2-1) \end{cases} \Rightarrow \begin{cases} \dot{x_1}=(c_3-1)x_1, \\ \dot{x_2}=0, \\ \dot{y_1}=x_1+c_3x_2-c_3, \\ \dot{y_2}=(1-c_3)x_2-(1-c_3). \end{cases}$$

$y_1^1=y_1+c_3\ ,y_2^1=y_2+(1-c_3)$ o'zgaruvchilar kiritsak,

$$\begin{cases} \dot{x_1}=(c_3-1)x_1, \\ \dot{x_2}=0, \\ \dot{y_1}=x_1+c_3x_2, \\ \dot{y_2}=(1-c_3)x_2. \end{cases}$$

Qulaylik uchun oldingi o'zgaruvchilar bilan yozib olamiz.

$$\begin{cases} \dot{x_1}=(c_3-1)x_1 \\ \dot{x_2}=0 \\ \dot{y_1}=x_1+c_3x_2 \\ \dot{y_2}=(1-c_3)x_2 \end{cases} \qquad A=\begin{pmatrix} c_3-1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & c_3 & 0 & 0 \\ 0 & 1-c_3 & 0 & 0 \end{pmatrix}$$

Bu matritsaning xos sonlari 3-betda topilgan.

$M_2^{13}(c_1,1-c_1;1{,}0)$ nuqta atrofida sistemani chiziqlashtiramiz:

$$\frac{\partial f_1}{\partial x_1}=y_1-1\big|M_2^{13}=0, \qquad \frac{\partial f_1}{\partial x_2}=0, \qquad \frac{\partial f_1}{\partial y_1}=x_1\big|M_2^{13}=c_1, \qquad \frac{\partial f_1}{\partial y_2}=0$$

$$\frac{\partial f_2}{\partial x_1} = \frac{\partial f_2}{\partial x_2} = \frac{\partial f_2}{\partial y_1} = \frac{\partial f_2}{\partial y_2} = 0$$

$$\frac{\partial f_3}{\partial x_1} = 1, \quad \frac{\partial f_3}{\partial x_2} = y_1 \big| M_2^{13} = 1, \quad \frac{\partial f_3}{\partial y_1} = x_2 - 1 \big| M_2^{13} = -c_1, \quad \frac{\partial f_3}{\partial y_2} = 0$$

$$\frac{\partial f_4}{\partial x_1} = 0, \quad \frac{\partial f_4}{\partial x_2} = y_2 \big| M_2^{13} = 0, \quad \frac{\partial f_4}{\partial y_1} = 0, \quad \frac{\partial f_4}{\partial y_2} = x_2 - 1 \big| M_2^{13} = -c_1$$

$$\begin{cases} \dot{x_1} = c_1(y_1 - 1) \\ \dot{x_2} = 0 \\ \dot{y_1} = x_1 - c_1 + x_2 - (1 - c_1) - c_1(y_1 - 1) \\ \dot{y_2} = -c_1 y_2 \end{cases} \Rightarrow$$

$$\begin{cases} \dot{x_1} = c_1 y_1 - c_1 \\ \dot{x_2} = 0 \\ \dot{y_1} = x_1 + x_2 - c_1 y_1 - 1 + c_1 \\ \dot{y_2} = -c_1 y_2 \end{cases} \Rightarrow$$

$c_1 y_1 - c_1 = y_1^1$, $+x_2 - 1 = x_2^1$ belgilash kiritamiz. $\dot{y_1} = \frac{1}{c_1}\dot{y}_1^1$

$$\begin{cases} \dot{x_1} = y_1^1 \\ -\dot{x}_2^1 = 0 \\ \frac{1}{c_1}\dot{y}_1^1 = x_1 + x_2^1 - y_2^1 \\ \dot{y_2} = -c_1 y_2 \end{cases} \Rightarrow A = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ c_1 & -c_1 & -c_1 & 0 \\ 0 & 0 & 0 & -c_1 \end{pmatrix}.$$

$$det|A - \lambda E| = \begin{vmatrix} -\lambda & 0 & 1 & 0 \\ 0 & -\lambda & 0 & 0 \\ c_1 & c_1 & -c_1 - \lambda & 0 \\ 0 & 0 & 0 & -c_1 - \lambda \end{vmatrix} =$$

$$= -\lambda \begin{vmatrix} -\lambda & 0 & 0 \\ c_1 & -c_1 - \lambda & 0 \\ 0 & 0 & -c_1 - \lambda \end{vmatrix} + \begin{vmatrix} 0 & -\lambda & 0 \\ c_1 & c_1 & 0 \\ 0 & 0 & -c_1 - \lambda \end{vmatrix} =$$

$$= \lambda^2(c_1 + \lambda)^2 - c_1\lambda(c_1 + \lambda) = \lambda(c_1 + \lambda)(\lambda(c_1 + \lambda) - c_1) = 0$$

$\lambda_1 = 0$, $\lambda_1 = -c_1$, $\lambda^2 + c_1\lambda - c_1 = 0$

$$\lambda_1 = 0,\ \lambda_2 = -c_1,\ \lambda_3 = \frac{-c_1-\sqrt{c_1^2+4c_1}}{2},\lambda_4 = \frac{-c_1+\sqrt{c_1^2+4c_1}}{2}$$

Bu yerda $\lambda_4 > 0$ bo'lganligi uchun, sistema $M_2^{13}(c_1, 1-c_1; 1,0)$ nuqta atrofida noturg'un.

Endi $M_3^{13}(0,1;1,0)$ nuqta atrofida sistemani o'rganamiz.

$$\frac{\partial f_1}{\partial x_1} = y_1 - 1\big|M_3^{13} = 0,\qquad \frac{\partial f_1}{\partial x_2} = 0,\qquad \frac{\partial f_1}{\partial y_1} = x_1\big|M_3^{13} = 0,\qquad \frac{\partial f_1}{\partial y_2} = 0.$$

$$\frac{\partial f_2}{\partial x_1} = \frac{\partial f_2}{\partial x_2} = \frac{\partial f_2}{\partial y_1} = \frac{\partial f_2}{\partial y_2} = 0$$

$$\frac{\partial f_3}{\partial x_1} = 1,\qquad \frac{\partial f_3}{\partial x_2} = y_1\big|M_2^{13} = 1,\qquad \frac{\partial f_3}{\partial y_1} = x_2 - 1\big|M_2^{13} = 0,\qquad \frac{\partial f_3}{\partial y_2}\big|M_2^{13} = 0$$

$$\frac{\partial f_4}{\partial x_1} = 0,\qquad \frac{\partial f_4}{\partial x_2} = y_2\big|M_3^{13} = 0,\qquad \frac{\partial f_4}{\partial y_1} = 0,\qquad \frac{\partial f_4}{\partial y_2} = x_2 - 1\big|M_3^{13} = 0$$

$$\begin{cases} \dot{x_1} = 0 \\ \dot{x_2} = 0 \\ \dot{y_1} = x_1 + x_2 - 1 \\ \dot{y_2} = 0 \end{cases}$$ $\Rightarrow\ x_2 - 1 = x_2^1$ deb olsak, Sistema quyidagi holga keladi (eski o'zgaruvchilarga qaytamiz):

$$\begin{cases} \dot{x_1} = 0 \\ \dot{x_2} = 0 \\ \dot{y_1} = x_1 + x_2 \\ \dot{y_2} = 0 \end{cases}$$

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \Rightarrow det|A - \lambda E| = \begin{vmatrix} -\lambda & 0 & 0 & 0 \\ 0 & -\lambda & 0 & 0 \\ 1 & 1 & -\lambda & 0 \\ 0 & 0 & 0 & -\lambda \end{vmatrix} = \lambda^4 = 0,$$

$$\lambda_{1,2,3,4} = 0$$

Demak bu sistemaning noturg'unligini aniqlab bo'lmaydi.

Endi $c_1 = 1$ bo'lsin $M_3^{13}(0,1;1,0)$ bo'ladi. Ushbu holda $\lambda_1 = 0$, $\lambda_2 = -c_1$, $\lambda_3 = \frac{-1-\sqrt{5}}{2}$, $\lambda_4 = \frac{-1+\sqrt{5}}{2}$ bo'lib, $\lambda_4 > 0$, demak Sistema noturg'un

Ushbu usulda Sistemaning turg'unligini topa olmadik. Endi ikkinchi usulda (1)-(4) sistemani ko'rib chiqamiz.

(3) dan

$\dot{y_1} = x_1 + (1 - x_1)y_1 - y_1 = x_1 + y_1 - x_1y_1 - y_1 = x_1 - x_1y_1 = x_1(1 - y_1)$

(4) dan $-\dot{y_1} = (x_2 - 1)y_2 = -x_1(1 - y_1) \Rightarrow$

$$\begin{cases} \dot{x_1} = x_1(y_1 - 1), \\ \dot{y_1} = x_1(1 - y_1). \end{cases}$$

Qo'zg'almas nuqtalar

$$M_1(0, c_3), M_2(c_1, 1), M_1^{13}(0,1; c_3, 1 - c_3), M_2^{13}(c_1, 1 - c_3; 1,0).$$

$V = x_1 + y_1$ Lyapunov funksiyasi bo'lsin.

$\dot{V} = x_1y_1 - x_1 + x_1 - x_1y_1 = 0 \Rightarrow$ Sistema turg'un.

FOYDALANILGAN ADABIYOTLAR RO'YXATI (REFERENCES)

1. Lyubich Yu.I. Mathematical Structures in Populatsion Genetics, Biomathematiks, 22, Springer-Verlag, Berlin, 1992.

2. Розиков У.А., Жамилов У.У. F-квадратичные стохастические операторы //Математические заметки, 83:4 (2008), с. 606-612

3. Розиков У.А., Жамилов У.У. Вольтерровские КСО двуполой популяции // Украинский математический журнал. 63:17 (2011). С. 985-998.

4. Rasulov Kh.R.«On a continuous time F - quadratic dynamical system». Uzbek mathematical journal, 2018 y., No4, p.126-130.

5. Расулов Х.Р., Яшиева Ф.Ю. О некоторых вольтерровских квадратичных стохастических операторах двуполой популяции с непрерывным временем // Наука, техника и образование, 77:2-2 (2021) с.23-26.

6. Расулов Х.Р., Раупова М.Х. Яшиева Ф.Ю. Икки жинсли популяция ва унинг математик модели ҳақида // Science and Education, scientific journal, 2:10 (2021), p.81-96.

7. Мафтуна Ҳошим қизи Ҳикматова. Квадратик стохастик операторлар қўлланиши ҳақида // Science and education, scientific journal 5:4 (2024), p 251-261.

8. Rasulov X. R. On a Volterra dynamical system of a two-sex population, Lobachevskii Journal of Mathematics, 45(8), 3975-3985 pp., 2024.

9. Расулов Х.Р. О качественном анализе одного класса вольтеровских квадратично-стохастических операторов с непрерывным временем. Математические заметки, 2024, том 116, выпуск 5, стр. 792–808.

10. Расулов Х.Р. Об одной квадратичной динамической системе с непрерывным временем // Тезисы международной научно-практической конференции «Актуальные задачи математического моделирования и информационных технологий» Nukus, May 2-3, 2023, Стр.286-287.

11. Расулов Х.Р. Аналог задачи Трикоми для квазилинейного уравнения смешанного типа с двумя линиями вырождения // Вестн. Сам. гос. техн. ун-та. Сер. Физ.-мат. науки, 2022. Т. 26, № 4.

12. Rasulov X.R. Qualitative analysis of strictly non-Volterra quadratic dynamical systems with continuous time // Communications in Mathematics, 30 (2022), no. 1, pp. 239-250.

13. Rasulov, H. (2021). One dynamic system with continuous time. Центр научных публикаций (buxdu.Uz), 5(5).

14. Xaydar Raupovich Rasulov. Boundary value problem in a domain with deviation from the characteristics for one nonlinear equation with mixed type. AIP Conf. Proc. 2781, 020016 (2023)

15. Расулов Х.Р. О некоторых символах математического анализа // Science and Education, scientific journal, 2:11 (2021), p.66-77.

16. Расулов Х.Р. О понятие асимптотического разложения и ее некоторые применения // Science and Education, scientific journal, 2:11 (2021), pp.77-88.

17. Xaydar R. Rasulov. On the solvability of a boundary value problem for a quasilinear equation of mixed type with two degeneration lines // Journal of Physics: Conference Series 2070 012002 (2021), pp.1–11.

18. Rasulov Kh.R. (2018). On a continuous time F - quadratic dynamical system // Uzbek Mathematical Journal, №4, pp.126-131.

19. Расулов Х.Р., Раупова М.Х. Роль математики в биологических науках // Проблемы педагогики, № 53:2 (2021), с. 7-10.

20. Rasulov, X. (2021). Краевая задача для одного нелинейного уравнения смешанного типа. Центр научных публикаций (buxdu.Uz), 7(7).

21. Rasulov, R. X. R. (2021). Гиперболик типдаги тенглама учун Коши масаласи. Центр научных публикаций (buxdu.Uz), 7(7).

22. Rasulov, R. X. R. (2022). О краевых задачах для уравнений эллиптического типа с линией искажения. Центр научных публикаций (buxdu.Uz), 8(8).

23. Шукурова М.Ф., Раупова М.Х. Каср тартибли интегралларни ҳисоблашга доир методик тавсиялар // Science and Education, scientific journal, 3:3 (2022), 65-76 b.

ISSN 2181-3187

24. Бозорова Д.Ш., Раупова М.Х. О функции Грина вырождающегося уравнения эллиптического типа // Science and Education, scientific journal, 3:3 (2022), с.14-22.

25. Жамолов Б.Ж., Раупова М.Х. О функции Римана вырождающегося уравнения гиперболического типа // Science and Education, scientific journal, 3:3 (2022), с.23-30.