BERNULLI DIFFERENSIAL TENGLAMALARNI O`QITISH METODIKASI

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Annotatsiya: Hozirgi kunda matematikaning differensial tenglamalar bo'limi juda rivojlanmoqda. Ta'lim sohasida esa alohida e'tibor qaratilmoqda shu bilan birgalikda differensial tenglamalar orqali ko'pgina masalalar o`z yechimini topmoqda. Differensial tenglamalarga oid masalalarni yechishda turli sohalarda keng qo'llanilmoqda. Masalan: ta'lim, tibbiyot , qurulish va boshqalar. Differensial tenglamaning Bernulli hamda chiziqli ko'rinishlari bilan differensial tenglamalar faniga kirib boramiz va ularni yechishni o'rganamiz.

Kalit so`zlar: Differensial tenglamalar, chiziqli differensial tenglamalar, μ-kiritish usuli, Bernulli, oddiy differensial tenglamalar, bir jinsli differensial tenglamalar, kvadraturalar.

Annotation: Currently, the field of differential equations in mathematics is rapidly developing. Special attention is being given to this area in education, and simultaneously, many problems are being solved through differential equations. Problems related to differential equations are widely applied in various fields such as education, medicine, construction, and others. We will explore the Bernoulli and linear forms of differential equations and learn how to solve them.

Keywords: Differential equations, linear differential equations, μ -integrating factor method, Bernoulli, ordinary differential equations, homogeneous differential equations, quadratures.

Аннотация: В настоящее время раздел математики — дифференциальные уравнения — активно развивается. В сфере образования этому уделяется особое внимание, и одновременно с этим многие задачи решаются с помощью дифференциальных уравнений. Задачи, связанные с дифференциальными уравнениями, широко применяются в различных областях, таких как образование, медицина, строительство и другие. Мы рассмотрим уравнения Бернулли и линейные дифференциальные уравнения и научимся их решать.

Ключевые слова: Дифференциальные уравнения, линейные дифференциальные уравнения, $\mu -$ метод интегрирующего множителя,

Бернулли, обыкновенные дифференциальные уравнения, однородные дифференциальные уравнения, квадратуры.

Kirish

Bernulli tenglamasi

1-Ta'rif. Ushbu

$$\frac{dy}{dx} = p(x)y + q(x)y^{\alpha} \qquad (1)$$

tenglama Bernulli tenglamasi deyiladi. Bu tenglamada berilgan $p(x)$ va $q(x)$ lar biror I intervalda aniqlanga funksiyalar. $\alpha -$ biror haqiqiy son $(\alpha \in R)$. Ravshanki, agar $\alpha = 0$ bo'lsa,

$$\frac{dy}{dx} = p(x)y + q(x)$$

chiziqli differensial tenglamaga ega bo'lamiz, agar $\alpha = 1$ bo'lsa, o`zgaruvchilari ajraladigan differensial tenglama hosil bo`ladi.

$$\frac{dy}{dx} = \big(p(x) + q(x)\big)y$$

Demak Bernulli tenglamasi $\alpha = 0, \alpha = 1$bo`lganida bizga ma'lum differensial tenglamalarga aylanadi Endi $\alpha \neq 0, \alpha \neq 1$ deb faraz qilamiz.

Asosiy qism

1-Teorema. Agar $p(x), q(x)$ funksiyalar I intervalda aniqlangan va uzluksiz bo'lib, $\alpha > 1$ bo`lsa, $G = \{\, (x, y) : x \in I, -\infty < y < +\infty\}$ sohaning $\forall(x_0, y_0)$ nuqtasidan (1) tenglamaning $I$ intervalda aniqlangan bitta integral chizig`i o'tadi.

U holda (1) tenglamani yechish uchun tenglikning har ikki tomonini $y \neq 0$, y^{α} ifodaga bo'lamiz. Ya'ni

$$\frac{y'}{y^{\alpha}} = \frac{p(x)}{y^{\alpha-1}} + q(x) \qquad (2)$$

(2) tenglamani hosil qilamiz , bu yerda $y' = \frac{dy}{dx}$.

(2) tenglamadan

$$\frac{1}{y^{\alpha-1}} = z(x)$$

almashtirish bajaramiz:

$$z' = (1 - \alpha)\frac{y'}{y^{\alpha}}$$

$$\frac{1}{1-\alpha}z' = p(x)z + q(x) \qquad (3)$$

Bu (3) tenglama. z ga nisbatan chiziqli differensial tenglama. Uning umumiy yechimi

$$z = \left(C + \int e^{-\int(1-\alpha)p(x)dx}(1-\alpha)q(x)dx\right)e^{\int(1-\alpha)p(x)dx}$$

ko'rinishda yoziladi.

Misol. Quydagi Bernulli tenglamasini yeching.

$xy' + 2y + x^5y^3e^x = 0.$ Bunda $C = 1.$

Yechish:

$$xy' + 2y + x^5y^3e^x = 0.$$

$$y' + \frac{2y}{x} + x^4y^3e^x = 0.$$

$$\frac{y'}{y^3} + \frac{2}{xy^2} = -x^4e^x.$$

$$|\frac{1}{y^2} = z \quad z' = -\frac{2y\prime}{y^3}|.$$

$$-\frac{t'}{2} + \frac{2t}{x} = -x^4e^x.$$

$$t' - \frac{4t}{x} = 2x^4e^x.$$

$$\mu(x) = e^{\int\frac{-4}{x}dx} = \frac{1}{x^4}.$$

$$\int d\left(\frac{t}{x^4}\right) = 2\int e^x dx.$$

$$\frac{t}{x^4} = 2(e^x + C).$$

$$\frac{1}{y^2} = 2x^4(e^x + C).$$

$$2x^4y^2(e^x + C) = 1.$$

$$C = 1 \; bunda \; y^2 = \frac{1}{2x^4(e^x+1)} .$$

Demak berilgan differensial tenglamaning umumiy yechimini topdik.

Endi berilgan misolni Maple dasturida ishlaymiz.:

```
> Bernoulli_ode :=diff(y(x),x)+(2*y(x))/x+x^4*y(x)^3*exp(x);
```

$$Bernoulli\_ode := \left(\frac{d}{dx}y(x)\right) + \frac{2\,y(x)}{x} + x^4\,y(x)^3\,e^x$$

```
> with(DEtools,odeadvisor);
```

[odeadvisor]

```
> odeadvisor(Bernoulli_ode);
```

[_Bernoulli]

```
> with(PDEtools,dchange);
```

[dchange]

```
> ITR := {y(x)=u(t)^(1/(1-3)),x=t};
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$$ITR := \{ y(x) = \frac{1}{\sqrt{u(t)}}, x = t \}$$

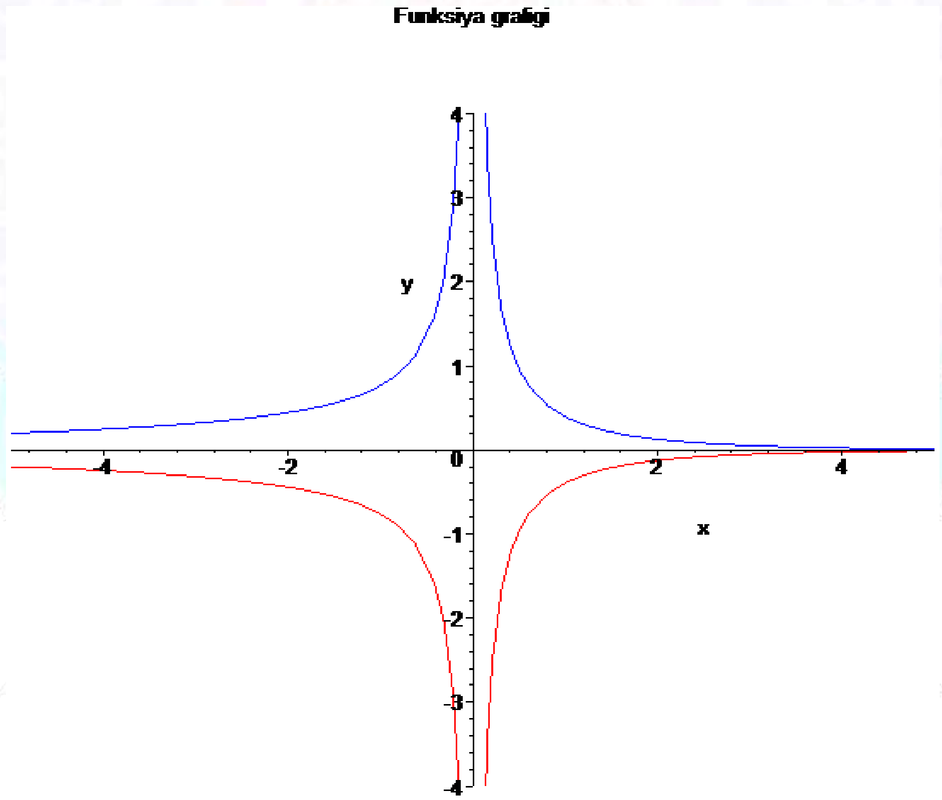
> new_ode := dchange(ITR,Bernoulli_ode,[u(t),t]):
new_ode2 := solve(new_ode,{diff(u(t),t)}):
op(factor(combine(expand(new_ode2),power)));

$$\frac{d}{dt} u(t) = \frac{2(2 u(t) + t^5 e^t)}{t}$$

> ans := dsolve(Bernoulli_ode);

$$ans := y(x) = \frac{1}{\sqrt{2 e^x + \_C1\ x^2}}, y(x) = -\frac{1}{\sqrt{2 e^x + \_C1\ x^2}}$$

Natijada $C = 1$ deb olsak, funksiya grafigi quyigicha bo'ladi:
> with(plots):
> Y1:=plot(((-1)*sqrt(1/(x^2*(1+2*exp(x))))), x=-5..5, y=-4..4, style=line,color=red):
Y2:=plot((sqrt(1/(x^2*(1+2*exp(x))))), x=-5..5, y=-4..4, style=line,color=blue):
display({Y1, Y2}, axes=boxed, scaling=constrained, title=`Funksiya grafigi`);



trained, title=`Funksiya grafigi`);

Xulosa: Bernulli tenglamasini ishlash uchun ma'lum bir algoritm yo`q. Rikk tenglamasiga yechim qiladiganda ketma- ket ishlash natijasida Bernulli , chiziqli, oddiy differensial tenglamalarga duch kelamiz. Maple 9.5 dasturidan foydalanib, berilgan differensial tenglamaga yechim qildik va yechimlarni grafiklari bilan ham tanishdik.

Foydalanilgan adabiyotlar ro`yxati:

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