TARKIBIDA TRIGONOMETRIK FUNKSIYALAR QATNASHGAN IFODALARNI INTEGRALLASH.

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Annotatsiya: Biz bu maqolada tarkibida trigonometrik funksiyalar qatnashgan ifodalarni sodda ko`rinishda integralashni o`rgamiz. Ya`ni bu maqolada trigonometrik funksiyarlarni belgilash kiritish yo`li bilan qulay usulda ishlashni o`rganamiz.Bundan tashqari maple dasturida trigonometrik ifodalarning grafigini chiqarishni ham o`rganamiz.

Kalit so`zlar: Ratsional funksiya,integral,trigonometrik funksiya,toq funksiya,juft funksiya.

Аннотация: В данной статье рассматривается интегрирование выражений, содержащих тригонометрические функции, в упрощённом виде. Мы изучаем удобные методы работы с тригонометрическими функциями с помощью введения обозначений. Кроме того, рассматривается построение графиков тригонометрических выражений с использованием программы Maple.

Ключевые слова: рациональная функция, интеграл, тригонометрическая функция, нечётная функция, чётная функция.

Abstract: In this article, we learn to integrate expressions involving trigonometric functions in a simplified form. Specifically, we explore convenient methods of working with trigonometric functions by introducing appropriate notations. Additionally, we study how to plot graphs of trigonometric expressions using the Maple software.

Keywords: rational function, integral, trigonometric function, odd function, even function.

Hamma trigonometrik funksiyalarni integrallash mumkin.Bu ifodani $R(sinx, cosx)$orqali belgilaymiz.Endi $R(sinx, cosx)$ ko`rinishidagi ifodani integrallaymiz.$\int R(sinx, cosx)dx$

Bunday integralni $tg\frac{x}{2} = t$ belgilash yordamida t o`zgaruvchili ratsional funksiyaning integraliga almashtirish mumkin.Integralni bunday almashtirish ratsionallashtirish deyiladi. Ko`rinib turibdiki, $tg\frac{x}{2} = t$ desak,

$sinx = \frac{2tg\frac{x}{2}}{1+tg^2\frac{x}{2}} = \frac{2t}{1+t^2}$; $cosx = \frac{1-tg^2\frac{x}{2}}{1+tg^2\frac{x}{2}} = \frac{1-t^2}{1+t^2}$; $\frac{x}{2} = arctgt \gg x = 2arctgt \gg dx = \frac{2dt}{1+t^2}$

Shuning uchun $\int R(sinx, cosx)dx = \int R\left(\frac{2t}{1+t^2}; \frac{1-t^2}{1+t^2}\right)\frac{2dt}{1+t^2} = \int R_1(t)dt$

bunda $R_1(t) - t$ o`zgaruvchili ratsional funksiya.Bunday almashtirish $R(sinx, cosx)$ ko`rinishidagi har qanday funksiyani integrallashga imkon beradi, shuning uchun bunday almashtirish ko`pincha ancha murakkab ratsional funksiyaga olib keladi.

1^0.Agar $R(sinx, cosx)$ ifoda $''sinx''$ ga nisbatan toq funksiya,ya`ni $R(-sinx, cosx) = -R(sinx, cosx)$ bo`lsa, u holda $''casx = t''$ $x \in (0; \pi)$ almashtirish bajarilsa $(1; 1)$ integral ostidagi ifoda t ning ratsional funksiyasiga keltiriladi.

1-misol.$\int sin^3x \cdot cos^3xdx = \left| \begin{matrix} cosx = t & x = arccost \\ sin^3x = \sqrt{(1-cos^2x)^3} & dx = -\frac{dt}{\sqrt{1-t^2}} \end{matrix} \right|$

$= \int \frac{\sqrt{(1-t^2)^3}t^2(-1)dt}{\sqrt{1-t^2}} = \int (1-t^2)t^2dt = \frac{t^5}{5} - \frac{t^3}{3} = \frac{1}{5}cos^5x - \frac{1}{3}cos^3x + C$

```
> f:=(((sin(x))^3)*((cos(x))^3));
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$f := \sin(x)^3 \cos(x)^3$

```
> Int (f,x);
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$\int \sin(x)^3 \cos(x)^3 \, dx$

```
> Int(f,x)=int (f,x);
> F:plot((((sin(x))^3)*((cos(x))^3)), x=-Pi..Pi, y=-Pi..Pi);
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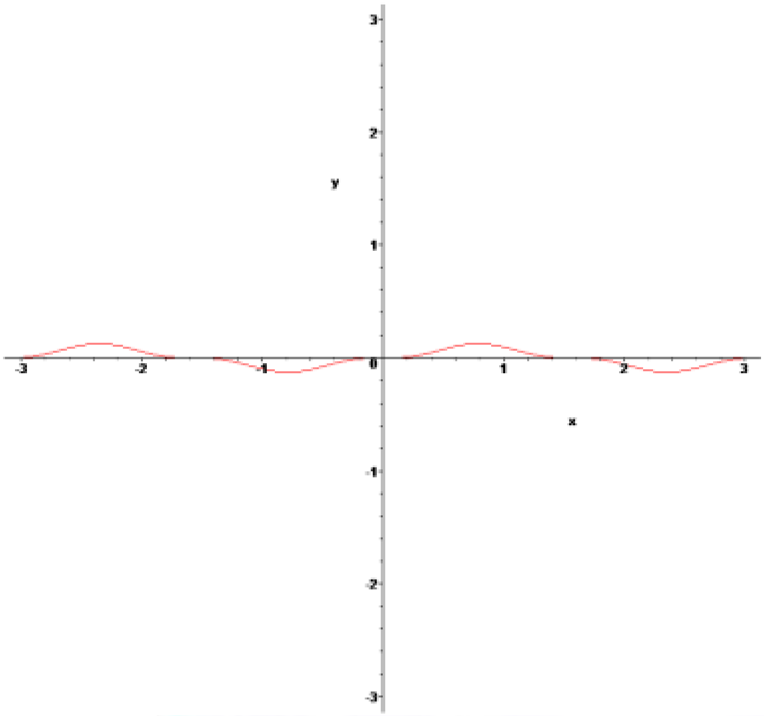
$\int \sin(x)^3 \cos(x)^3 \, dx = -\frac{1}{6}\sin(x)^2\cos(x)^4 - \frac{1}{12}\cos(x)^4$



Demak, bu integralning chizmasi yuqoridagi ko`rinishda ekan.

2^0. Agar $R(-sinx, -cosx) = R(sinx, cosx)$ bo`lsa, u holda $t = tgx$

$x \in \left(\frac{\pi}{2}; -\frac{\pi}{2}\right)$ almashtirishdan biri bajariladi.

2-misol. $\int \frac{3sinx-2cosx}{1+cosx}dx = \left| \begin{matrix} tg\frac{x}{2}=t \; x=2arctgt \; cosx=\frac{1-t^2}{1+t^2} \\ dx=\frac{2}{1+t^2} \quad sinx=\frac{2t}{1+t^2} \end{matrix} \right| = \int \frac{2(3\frac{2}{1+t^2}-2\frac{1-t^2}{1+t^2})}{(1+\frac{1-t^2}{1+t^2})(1+t^2)}dt =$

$2\int \frac{t^2+1-2+8t}{1+t^2}dt \quad = 2\int dt - 4\int \frac{dt}{1+t^2} + 6\int \frac{d(1+t^2)}{1+t^2} = 2t - 4arctgt +$

$+3ln|1+t^2| + C = 2tg(\frac{x}{2}) + 3\ln\left(tg\left(\frac{x}{2}\right)^2 + 1\right) - 2x + C;$

Endi bu misolni yechimini va grafigini maple dasturida ko`ramiz:

> f:=((3*sin(x)-2*cos(x))/(1+cos(x)));

$$f := \frac{3\sin(x) - 2\cos(x)}{1+\cos(x)}$$

> Int(f,x);

$$\int \frac{3\sin(x) - 2\cos(x)}{1+\cos(x)}\,dx$$

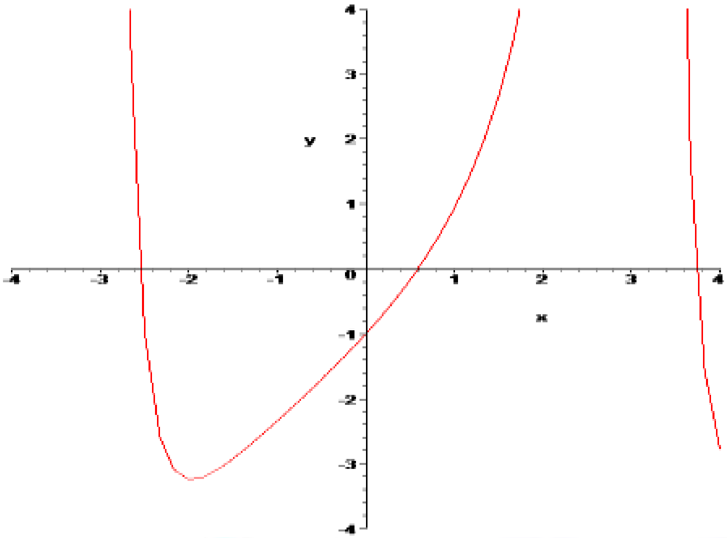
> int(f,x);

$$2\tan\left(\frac{x}{2}\right) + 3\ln\left(\tan\left(\frac{x}{2}\right)^2 + 1\right) - 2x$$

> Int(f,x)=int(f,x);

$$\int \frac{3\sin(x) - 2\cos(x)}{1+\cos(x)}\,dx = 2\tan\left(\frac{x}{2}\right) + 3\ln\left(\tan\left(\frac{x}{2}\right)^2 + 1\right) - 2x$$

> plot(((3*sin(x)-2*cos(x))/(1+cos(x))), x=-4..4, y=-4..4);



Demak, bu integralning chizmasi yuqoridagi ko`rinishda ekan.

3^0. $\int sin\alpha \cdot xcos\beta x\,dx$, $\int sin\alpha \cdot xsin\beta x\,dx$, $\int cos\alpha x \cdot cos\beta x\,dx$, ko`rinishidagi integrallarni hisoblash.

Foydalanuvchi formulalar:

1) $sin\alpha x \cdot cos\beta x = \frac{1}{2}[\sin(\alpha+\beta)\,x + \sin(\alpha-\beta)x]$;

2) $sin\alpha x \cdot sin\beta x = \frac{1}{2}[\cos(\alpha-\beta)x - \cos(\alpha+\beta)\,x]$;

3) $cos\alpha x \cdot cos\beta x = \frac{1}{2}[\cos(\alpha+\beta)x + \cos(\alpha-\beta)\,x]$;

3-misol. $\int cos4x \cdot cosxdx = \frac{1}{2}(cos5x - cos3x)dx = \frac{1}{2}\int cos5x\,dx + \frac{1}{2}\int cos3x\,dx = \frac{1}{10}sin5x + \frac{1}{6}sin3x + C.$

> **f:=((cos(4*x)*cos(x)));**

$f := \cos(4\,x)\cos(x)$

> **Int(f,x);**

$$\int \cos(4\,x)\cos(x)\,dx$$

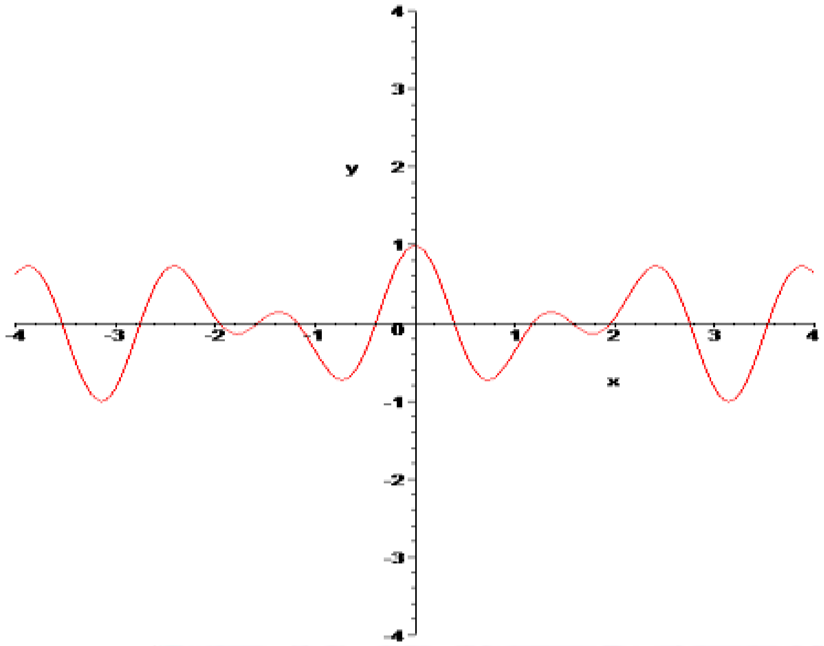
> **int(f,x);**

$$\frac{1}{6}\sin(3\,x) + \frac{1}{10}\sin(5\,x)$$

> **Int(f,x)=int(f,x);**

$$\int \cos(4\,x)\cos(x)\,dx = \frac{1}{6}\sin(3\,x) + \frac{1}{10}\sin(5\,x)$$

> **plot(((cos(4*x)*cos(x))), x=-4..4, y=-4..4);**



Demak, bu integralning chizmasi yuqoridagi ko`rinishda ekan.

4^0. $\int sin^m x \cdot cos^n x dx \ (n, m \in z)$ ko`rinishidagi integrallarni hisoblash.

$I.$ n, m lar manfiy bo`lmagan $(n, m \in z)$ $n > 0, m > 0$ juft son bo`lgan holat.Bu holatda darajani pasaytirish, ya`ni

$$sin^2 x = \frac{1}{2}(1 - cosx) \quad cos^2 x = \frac{1}{2}(1 + cos2x)$$

Formula qo`llaniladi.

4-misol.$\int sin^4 x \cdot cos^2 x dx = \frac{1}{4}\int sin^2 x \cdot sin^2 2x dx = \frac{1}{4}\int \frac{1-cos2x}{2} \cdot \frac{1-cos4x}{2} dx = \frac{1}{16}\int (1 - cos4x - cos2x + cos2x \cdot cos4x) dx = \frac{1}{16}x - \frac{1}{64}sin4x - \frac{1}{32}sin2x + \frac{1}{16}\int (cos2x + cos6x) dx = \frac{1}{16}x - \frac{1}{64}sin2x - \frac{1}{64}sin4x + \frac{1}{182}sin6x + C.$

$II. n, m$ larning ikkalasi ham butun manfiy $n, m \in z$ va juft yoki toq bo`lganda $tgx = t$ yoki $ctgx = t$ olinib $1 + tg^2 x = \frac{1}{cos^2 x}$ $\ 1 + ctg^2 x = \frac{1}{sin^2 x}$ formuladan foydalaniladi.

5-misol.$\int \frac{1}{sin^3 x \cdot cos^6 x} dx = \int \frac{1}{sin^8 x \cdot \frac{cos^5 x}{sin^5 x}} = \int \frac{dx}{sin^2 x \cdot sin^6 x \cdot \frac{cos^5 x}{sin^5 x}} = -\int \frac{1}{ctg^5 x} \cdot (1 + +ctg^2 x)^3 d(ctgx) = \left| \begin{matrix} ctgx = t \\ dctgx = dt \end{matrix} \right| = \frac{1}{4}t^{-4} + \frac{3}{2}t^{-2} - 3\ln|t| - \frac{t^2}{2} + C = \frac{1}{4} \cdot \frac{1}{ctg^4 x} + \frac{3}{2} \cdot \frac{1}{ctg2x} - 3ln|ctgx| + \frac{ctg^2 x}{2} + C.$

```
> f:=(1/((sin(x))^3)*((cos(x))^6));
```

$$f := \frac{\cos(x)^6}{\sin(x)^3}$$

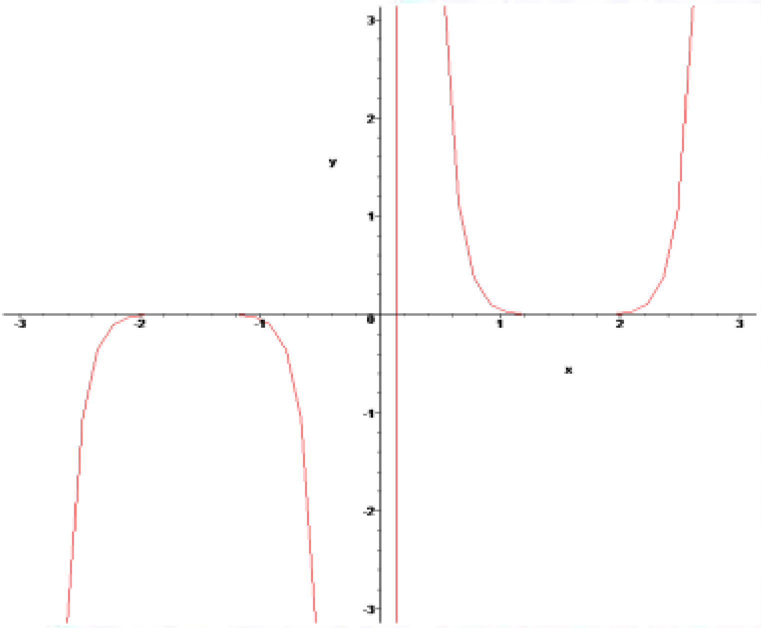
```
> Int (f,x);
```

$$\int \frac{\cos(x)^6}{\sin(x)^3}\, dx$$

> Int(f,x)=int (f,x);

> F:plot((1/((sin(x))^3)*((cos(x))^6)), x=-Pi..Pi, y=-Pi..Pi);

$$\int \frac{\cos(x)^6}{\sin(x)^3}\, dx = -\frac{1}{2}\frac{\cos(x)^7}{\sin(x)^2} - \frac{1}{2}\cos(x)^5 - \frac{5}{6}\cos(x)^3 - \frac{5}{2}\cos(x) - \frac{5}{2}\ln(\csc(x) - \cot(x))$$



Xulosa: Trigonometrik ifodalarni soda ko`rinishga keltirmasdan uni integrallash ancha murakkab ishligi sababli uni yechishni ossonlashtirdik.Ya`ni belgilash kiritish orqali integrallarga yechim topdik.Sodda ko`rinishga keltirilganligi sababli trigonometrik ifodalarni integralllash ancha oson bo`ladi. Maple 9.5 dasturidan foydalanib, berilgan trigonometrik ifodalarnni integrallashni yechim qildik va yechimlarni grafiklari bilan ham tanishdik.

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