OLMOS PANJARADAGI DISKRET SHRÖDINGER OPERATORNING MUHIM SPEKTRI

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Annotatsiya: Ushbu maqolada olmos panjaradagi diskret Shrödinger operatorining muhim spektri o'rganiladi. Tadqiqotda ushbu operatorning spektral xossalari, xususan, spektrning tarkibi, uning nuqtaviy va uzluksiz qismlari, shuningdek, bu operatorga bog'liq bo'lgan matritsali ifodalar tahlil qilinadi. Shuningdek, olmos panjaraga xos bo'lgan geometrik va topologik tuzilmalarning operator spektriga ta'siri ko'rib chiqiladi. Olingan natijalar kvant mexanikasi, kristall panjaralardagi elektron holatlarini modellashtirish, va matematik fizika sohalarida qo'llanilishi mumkin. Maqolada analitik metodlar bilan bir qatorda, ba'zi misollar orqali operator spektrining strukturaviy xususiyatlari ham ochib beriladi.

Kalit so'zlar: olmos panjara, regulyar nuqta, spektr, operatorning spektri, diskret spektr, muhim spektr, kompakt operator.

Quyidagi to'plamni qaraymiz:

$A_2 = \{v(n): \ v(n) = n_1 v_1 + n_2 v_2 \quad n = (n_1\,; n_2), \ n \epsilon \mathbb{Z}^2\},$

bu yerda $v_1 = (-1\,; 0; 1) \quad v_2 = (0\,; -1\,; 1)$.

Ta'rif 1. A_2 to'plamga 2 o'lchamli olmos panjara deyiladi.

Quyidagi to'plamni kiritamiz:

$\Omega = A_2 \cup (p + A_2)\,, \quad p = \frac{1}{3}(-1\,; -1; 2).$

$\ell_2(\Omega)$ - orqali Ω da kvadrati bilan jamlanuvchi $\hat{f}(n) = (\hat{f}_1(n), \hat{f}_2(n))$ funksiyalar juftligini belgilaymiz. Bu fazo Gilbert fazosi bo'lib, skalyar ko'paytma quydagicha aniqlangan

$$(\hat{f}, \hat{g}) = \sum_{v \epsilon A_2} 3\hat{f}_1(n)\hat{g}_1(n) + \sum_{v \epsilon (p + A_2)} 3\hat{f}_2(n)\hat{g}_2(n).$$

$\mathbb{T} = (-\pi\,; \pi]$. $L_2{}^{(2)}(\mathbb{T}^2) - \mathbb{T}^2$ da aniqlangan kvadrati bilan integrallanuvchi $\mathrm{f}(x) = (\mathrm{f}_1(x), \mathrm{f}_2(x))$ funksiyalar juftligining Gilbert fazosi bo`lsin. Bu yerda skalyar ko'paytma quydagicha aniqlangan:

$$(f, g) = (f_1, g_1) + (f_2, g_2)$$

bunda

$$(f_i, g_i) = \int\limits_{\mathbb{T}^2} f_i(x)\overline{g_i(x)}\,dx, \quad i = 1,2.$$

Quydagi $F : \ell_2(\Omega) \to L_2^{(2)}(\mathbb{T}^2)$ *unitar operatorni kiritamiz:*

$$F = \begin{pmatrix} \mathcal{F} & 0 \\ 0 & \mathcal{F} \end{pmatrix}, \qquad (\mathcal{F}\hat{f})(x) = \frac{\sqrt{3}}{2\pi}\sum_{n\in\mathbb{Z}^2} e^{i(x,s)}\hat{f}(s).$$

Bu operator teskarisi $F^{-1}: L_2^{(2)}(\mathbb{T}^2) \to \ell_2(\Omega)$ quydagicha aniqlanadi:

$$F^{-1} = \begin{pmatrix} \mathcal{F}^{-1} & 0 \\ 0 & \mathcal{F}^{-1} \end{pmatrix}, \ (\mathcal{F}^{-1}f)(s) = \frac{\sqrt{3}}{2\pi}\int_{\mathbb{T}^2} e^{-i(s,x)} f(x)dx.$$

bu yerda $(s, x) = s_1 x_1 + s_2 x_2$.

Olmos panjaradagi diskrit Shredinger operatori $\widehat{H}$ ushbu $\ell_2(\Omega)$ fazoda chegaralangan o‘z-o‘ziga qo‘shma operator sifatida quyidagicha aniqlanadi:

$$\widehat{H} = -3(\Delta_2 + 1) + \hat{Q}$$

bunda $\left(-3(\Delta_2 + 1)\hat{f}\right)(v) = \left((V_1\hat{f}_2)(n);\ (V_2\hat{f}_1)(n)\right),$

bu yerda $(V_1\hat{f}_2)(n) = \hat{f}_2(\mathrm{n}) + \hat{f}_2(\mathrm{n} - e_1) + \hat{f}_2(\mathrm{n} - e_2)$

$$(V_2\hat{f}_1)(n) = \hat{f}_1(\mathrm{n}) + \hat{f}_1(\mathrm{n} - e_1) + \hat{f}_1(\mathrm{n} - e_2)$$

$$e_1,\ e_2,\ n\ \epsilon\Omega, \qquad n = (n_1; n_2),\ \ e_1 = (1;0),\ \ e_2 = (0;1).$$

$\hat{Q}$- Ω da aniqlangan zarrachalarning o‘zaro ta’sir potensiali bo‘lib, ular quyidagi formulalar bilan aniqlanadi:

$$(\hat{Q}f)(n) = \begin{pmatrix} \hat{Q}_1(n) & 0 \\ 0 & \hat{Q}_2(n) \end{pmatrix}\begin{pmatrix} \hat{f}_1(n) \\ \hat{f}_2(n) \end{pmatrix} = \begin{pmatrix} \hat{Q}_1(n)\hat{f}_1(n) \\ \hat{Q}_2(n)\hat{f}_2(n) \end{pmatrix}$$

bunda

$$\sum_{n\in A_2} |\hat{Q}_1(n)| < \infty \ , \quad \sum_{n\in(p+A_2)} |\hat{Q}_2(n)| < \infty \ .$$

$\widehat{H}$ operatorni koordinata ko‘rinishidan impuls tasvirga o‘tish F almashtirishilari yordamida amalga oshiriladi.

$$H = F\widehat{H}F^{-1} = F\left(-3(\Delta_2 + 1)\right)F^{-1} + F\hat{Q}F^{-1}\ .$$

H operator olmos panjaradagi diskrit Shredinger operatorining impuls tasviri bo‘lib, u quydagicha aniqlanad:

$$H = H_0 + Q\ , \qquad (1)$$

bu yerda : H_0 va Q 2×2 matritsa uchun matritsa operatorlari bo‘lib, $L_2^{(2)}(\mathbb{T}^2)$ da quyidagicha aniqlanadi:

$$(H_0 f)(x) = \begin{pmatrix} 0 & E(x) \\ \overline{E(x)} & 0 \end{pmatrix}\begin{pmatrix} f_1(x) \\ f_2(x) \end{pmatrix} = \begin{pmatrix} E(x)f_2(x) \\ \overline{E(x)}f_1(x) \end{pmatrix},$$

$$(Qf)(x) = \begin{pmatrix} Q_1 & 0 \\ 0 & Q_2 \end{pmatrix}\begin{pmatrix} f_1(x) \\ f_2(x) \end{pmatrix} = \begin{pmatrix} (Q_1 f_1)(x) \\ (Q_2 f_2)(x) \end{pmatrix},$$

bunda, $E(x) -$ 2 o`zgaruvchili kompleks qiymatli funksiya $E(x) = \frac{1}{3}\left(1 + e^{ix_1} + e^{ix_2}\right)$, $Q_i - L_2(\mathbb{T}^2)$ da aniqlangan integral operator

$$(Q_i f_i)(x) = \int_{\mathbb{T}^2} Q_i(x-t) f_i(t)dt. \qquad i = 1, 2,$$

$Q_i(\cdot) -$ $\mathbb{T}^2$ da aniqlangan haqiqiy qiymatli biror uzluksiz, juft funksiya.

Mazkur maqolada $L_2^{(2)}(\mathbb{T}^2) -$ Gilbert fazosida (1) ko'rinishda aniqlangan Olmos panjaradagi diskret Shredinger operatori $H = H_0 + Q$ ning muhim spektrini o'rganamiz.

Bizga ma'lumki biror $\lambda \in \mathbb{C}$ uchun $A - \lambda I$ operator teskarilanuvchan bo'lsa, u holda λ soni A operatorning **regulyar nuqtasi** deb atalar edi va A operatorning barcha regulyar nuqtalari to'plami $\rho(A)$ kabi belgilagan edik.

Ta`rif 2. $\sigma(A) = \mathbb{C} \backslash \rho(A)$ *to'plam A operatorning* **spektri** *deb ataladi.*

Ta`rif 3. $\lambda \in \sigma(A)$ *son yakkalangan, A operatorning chekli karrali xos qiymatlari to'plami* **diskret spektr** *deb ataladi va* $\sigma_{disc}(A)$ *deb belgilanadi.*

Ta`rif 4. $\sigma_{ess}(A) = \sigma(A) \backslash \sigma_{disc}(A)$, *A* **muhim spektr** *deb ataladi.*

Ushbu tadqiqotning asosiy teoremasi quyidagidan iborat

Teorema 1. $\sigma(H_0) = [-1\,; 1]$.

Isbot. Bizga ma`lumki [Reed Simon], H_0 2×2 matritsa operatorining spektri quyidagi formula bilan aniqlanadi:

$$\sigma(H_0) = \bigcup_{x \in T^2} \sigma(H_0(x)). \qquad (*)$$

Bunda $H_0(x) -$ har bir fiksirlangan $x \in T^2$ da 2×2 sonli matritsa bo`ladi, ya'ni

$$H_0(x) = \begin{pmatrix} 0 & E(x) \\ \overline{E(x)} & 0 \end{pmatrix}, \qquad E(x) = \frac{1}{3}\left(1 + e^{ix_1} + e^{ix_2}\right).$$

Shuning uchun $H_0(x)$ ning spektri xos qiymatlaridan iborat bo'ladi, ya`ni har bir tayinlangan $x \in T^2$ larda $det|H_0 - \lambda I| = 0$ tenglamaning ildizlaridan iboratdir. Bu tenglamani tuzamiz:

$$(H_0 - \lambda I)(x) = \begin{pmatrix} 0 & E(x) \\ \overline{E(x)} & 0 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} = \begin{pmatrix} -\lambda & E(x) \\ \overline{E(x)} & -\lambda \end{pmatrix}.$$

$$det|H_0 - \lambda I| = \begin{vmatrix} -\lambda & E(x) \\ \overline{E(x)} & -\lambda \end{vmatrix} = 0 \implies \lambda^2 - E(x) \cdot \overline{E(x)} = 0 \implies$$

$$\lambda^2 = |E(x)|^2 \implies \lambda_{1,2} = \pm|E(x)|, \quad x \in T^2.$$

bu yerda

$$|E(x)|^2 = E(x)\overline{E(x)} = \frac{1}{3}\left(1+e^{ix_1}+e^{ix_2}\right)\frac{1}{3}\left(1+e^{-ix_1}+e^{-ix_2}\right) = \frac{1}{9}\Big(3+ e^{ix_1}+e^{-ix_1}+e^{ix_2}+e^{-ix_2}++e^{i(x_1-x_2)}+e^{-i(x_1-x_2)}\Big) =$$

$$= \frac{1}{9}(3+2\cos x_1+2\cos x_2+2\cos(x_1-x_2)).$$

Shunday qilib,

$\sigma(H_0(x)) = \{\, xos\ qiymatlari \,\} = \{-|E(x)|\,;\ |E(x)|\,\}.$

Demak, (*) ga ko‘ra

$$\sigma(H_0) = \cup_{x\in T^2}\sigma(H_0(x)) = \cup_{x\in T^2}\{-|E(x)|\,;\ |E(x)|\,\}$$

$= -Ran\{|E(x)|\} \ \cup\ Ran\{|E(x)|\}\,.$

Endi $|E(x)| = \max\limits_{x\in T^2}\frac{1}{9}(3+2\cos x_1+2\cos x_2+2\cos(x_1-x_2)) = 1$ va $\min\limits_{x\in T^2}|E(x)| = 0$ ekanligidan, ushbu $-Ran\{|E(x)|\} = [-1;0]$ va $Ran\{|E(x)|\} = [0;1]$ tengliklarni hosil qilamiz.

Demak $\sigma(H_0) = [-1\,;1]\,.$

Teorema isbotlandi.

Lemma 1. $Q\ :\ L_2^{(2)}(\mathbb{T}^2) \to L_2^{(2)}(\mathbb{T}^2)$ *kompakt operator*.

Isbot. $Q\ :\ L_2^{(2)}(\mathbb{T}^2) \to L_2^{(2)}(\mathbb{T}^2)$. operatorni ko`rinishi quydagicha edi:

$$(Qf)(x) = \begin{pmatrix}(Q_1f_1)(x)\\(Q_2f_2)(x)\end{pmatrix} = \begin{pmatrix}(\mu_1f_1)(x)\\(\mu_2f_2)(x)\end{pmatrix}$$

bunda, $(Q_if_i)(x) = \int_{\mathbb{T}^2} Q_i(x-t)f_i(t)dt.$ $i = 1,2,$ $\mu_1,\mu_2 > 0.$

Biz Q operatorni kompaktligini ko‘rsatishimiz uchun har bir $i\in\{1\,,2\}$ da $Q_i : L_2(\mathbb{T}^2) \to L_2(\mathbb{T}^2)$ operatorni kompakt ekanligini ko`rsatamiz. Ma’lumki, $(Q_if_i)(x) = \int_{\mathbb{T}^2} Q_i(x-t)f_i(t)dt.$ operator kompakt bo`lishi uchun $\int_{T^2}\int_{T^2}|Q_i(x-t)|^2dtdx < \infty$ bo`lishi zarur va yetarli. Shartga ko`ra $Q_i(\cdot)-$ ikkala o`zgaruvchi buyicha ham $\mathbb{T}^2$ da aniqlangan biror uzluksiz funksiya.

Bundan $\int_{T^2}\int_{T^2}|Q_i(x-t)|^2dtdx$ integral mavjud va chekli. Demak Q_i kampakt , ya`ni Q kampakt operator.

Teorema 2. $\sigma_{ess}(H) = \sigma(H_0) = [-1,1]\,.$

Isbot. Muhum spektr turg`unligi haqidagi ***Veyl teoremasi***_ ga ko`ra $H = Q + H_0$ operatorning muhim spektri Q kompakt qo`zg`alishda o`zgarmaydi va H_0 operator spektri bilan ustma-ust tushadi. Q kampakt operator. Bu yerdan esa xulosa $\sigma_{ess}(H) = \sigma(H_0)$.

Teorema isbotlandi.

Biz endi $H = H_0 + Q$ operatorning $Q = \mu_i > 0, i = 1{,}2$ holdagi xos qiymati va unga mos xos vektorini topish masalasini ko‘rib chiqamiz. Buning uchun quyidagi belgilashlarni kiritamiz:

$D(z) = \begin{vmatrix}\Delta_{11} & \Delta_{12}\\ \Delta_{21} & \Delta_{22}\end{vmatrix}$, ko‘rinishdagi $2x2$ matritsa, bunda

$$\begin{cases} \Delta_{11} = 1 + z\mu_1 \int\limits_{T^2} \frac{1}{|E(s)|^2 - z^2} ds, & \Delta_{12} = \mu_2 \int\limits_{T^2} \frac{E(s)}{|E(s)|^2 - z^2} ds \\ \Delta_{21} = \mu_1 \int\limits_{T^2} \frac{\overline{E(s)}}{|E(s)|^2 - z^2} ds, & \Delta_{22} = 1 + z\mu_2 \int\limits_{T^2} \frac{1}{|E(s)|^2 - z^2} ds \end{cases} \quad (1)$$

Teorema 3. $z \in R\backslash[-1,1]$ soni H operatorning xos qiymati bo‘lishi uchun $D(z) = 0$ bo‘lishi zarur va yetarli.

Isbot(Zaruriyligi): z sono H operatorning xos qiymati bo‘lsin, ya’ni

$$H\psi = z\psi$$
$$\psi = (\psi_1, \psi_2), \psi_1, \psi_2 \in L_2(T^2)$$

tenglik bajarilsin. H operator chiziqliligidan

$$H\psi = (H_0 + Q)\psi = H_0\psi + Q\psi = \begin{pmatrix} E(x) & \psi_2 \\ \overline{E(x)} & \psi_1 \end{pmatrix} + \begin{pmatrix} \int_{T^2} \mu_1 \psi_1(s)\, ds \\ \int_{T^2} \mu_2 \psi_2(s)\, ds \end{pmatrix} = z \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

Bu yerdan quyidagi tenglamalar sistemasini hosil qilamiz:

$$\begin{cases} E(x)\psi_2(x) + \mu_1 \int\limits_{T^2} \psi_1(s)\, ds = z\psi_1(x) \\ \overline{E(x)}\psi_1(x) + \mu_2 \int\limits_{T^2} \psi_2(s)\, ds = z\psi_2(x) \end{cases} \quad (2)$$

Quyidagicha belgilash kiritamiz

$$C_1 = \int\limits_{T^2} \psi_1(s)\, ds, \qquad C_2 = \int\limits_{T^2} \psi_2(s)\, ds \qquad (3)$$

U holda (1) ga (2) ni qo‘yib quyidagi sistemasiga ega bo‘lamiz:

$$\begin{cases} E(x)\psi_2(x) + \mu_1 C_1 = z\psi_1(x) \\ \overline{E(x)}\psi_1(x) + \mu_2 C_2 = z\psi_2(x) \end{cases}$$

bundan

$$\begin{cases} E(x)\psi_2(x) = -\mu_1 C_1 + z\psi_1(x) \\ \overline{E(x)}\psi_1(x) = -\mu_2 C_2 + z\psi_2(x) \end{cases} \quad (4)$$

ni hosil qilamiz. Dastlab birinchi va ikkinchi tenglamani mos rvishda $\overline{E(x)}$ va $E(x)$ larga ko‘paytiramiz:

$$\begin{cases} |E(x)|^2\psi_2(x) + \mu_1 \overline{E(x)} C_1 = z\overline{E(x)}\psi_1(x) \\ |E(x)|^2\psi_1(x) + \mu_2 E(x) C_2 = zE(x)\psi_2(x) \end{cases}$$

Oxirgi sistemaga (3) ni qo‘yamiz va quyidagi sistemani hosil qilamiz:

$$\begin{cases}|E(x)|^2\psi_2(x)+\mu_1\overline{E(x)}C_1=z\big(-\mu_2C_2+z\psi_2(x)\big)\\|E(x)|^2\psi_1(x)+\mu_2E(x)C_2=z\big(-\mu_1C_1+z\psi_1(x)\big)\end{cases}$$

yoki

$$\begin{cases}(|E(x)|^2-z^2)\psi_2(x)=-\mu_1\overline{E(x)}C_1-z\mu_2C_2\\(|E(x)|^2-z^2)\psi_1(x)=-\mu_2E(x)C_2-z\mu_1C_1\end{cases}$$

$|E(x)|^2-z^2\neq 0$ ekanligidan ψ_1,ψ_2 funksiyalarga ega bo‘lamiz:

$$\begin{cases}\psi_1(x)=-\dfrac{\mu_2E(x)}{|E(x)|^2-z^2}C_2-\dfrac{z\mu_1}{|E(x)|^2-z^2}C_1\\\psi_2(x)=-\dfrac{\mu_1\overline{E(x)}}{|E(x)|^2-z^2}C_1-\dfrac{z\mu_2}{|E(x)|^2-z^2}C_2\end{cases}\qquad(5)$$

Endi (5) ni (3) ga olib borib qo‘yamiz va quyidagini hosil qilamiz:

$$\begin{cases}C_1=\displaystyle\int_{T^2}\left(-\frac{\mu_2E(s)}{|E(s)|^2-z^2}C_2-\frac{z\mu_1}{|E(s)|^2-z^2}C_1\right)ds\\C_2=\displaystyle\int_{T^2}\left(-\frac{\mu_1\overline{E(s)}}{|E(s)|^2-z^2}C_1-\frac{z\mu_2}{|E(s)|^2-z^2}C_2\right)ds\end{cases}$$

$$\begin{cases}C_1\left(1+z\mu_1\displaystyle\int_{T^2}\frac{1}{|E(s)|^2-z^2}ds\right)+C_2\mu_2\displaystyle\int_{T^2}\frac{E(s)}{|E(s)|^2-z^2}ds=0\\C_1\mu_1\displaystyle\int_{T^2}\frac{\overline{E(s)}}{|E(s)|^2-z^2}ds+C_2\left(1+z\mu_2\displaystyle\int_{T^2}\frac{1}{|E(s)|^2-z^2}ds\right)=0\end{cases}$$

bu yerdan (1) ga ko‘ra

$$\begin{cases}C_1\Delta_{11}+C_2\Delta_{12}=0\\C_1\Delta_{21}+C_2\Delta_{22}=0\end{cases}$$

ko‘rinishdagi tenglamalar sistemasiga kelamiz. Bunda $\psi=(\psi_1,\psi_2)$ vektor H operatorning xos funksiyalari bo‘lganligi uchun $(C_1,C_2)\neq 0$ bo‘lishi lozim. Bizga algebra kursidan ma’lumki, bir jinsli tenglamalar sistemasi noldan farqli yechimga ega bo‘lishi uchun asosiy determinant nolga teng bo‘lishi zarur. Bundan

$$D(z)=\begin{vmatrix}\Delta_{11}&\Delta_{12}\\\Delta_{21}&\Delta_{22}\end{vmatrix}=0.$$

Yetarliligi: Faraz qilaylik, $z\in R\backslash[-1,1]$ ga $D(z)=\begin{vmatrix}\Delta_{11}&\Delta_{12}\\\Delta_{21}&\Delta_{22}\end{vmatrix}=0$ bo‘lsin. U holda shunday $(C_1,C_2)\neq 0$ mavjudki,

$$\begin{cases}\Delta_{11}C_1+\Delta_{12}C_2=0\\\Delta_{21}C_1+\Delta_{22}C_2=0\end{cases}$$

tengliklar o‘rinli bo‘ladi.

$\psi=(\psi_1,\psi_2)$ vektorda $\psi_1$va ψ_2 lar quyidagi ko‘rinishda bo‘lsin:

$$\psi_1(x) = -\frac{\mu_2 E(x)}{|E(x)|^2 - z^2} C_2 - \frac{z\mu_1}{|E(x)|^2 - z^2} C_1$$

$$\psi_2(x) = -\frac{\mu_1 \overline{E(x)}}{|E(x)|^2 - z^2} C_1 - \frac{z\mu_2}{|E(x)|^2 - z^2} C_2$$

$\psi = (\psi_1, \psi_2)$ ni H operatorning xos vektori ekanligini ko'rsatamiz. Buning uchun H operatorning $\psi = \begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$ vektorga ta'sirini qaraymiz.

$$H\psi = (H_0 + Q)\psi = \left(\begin{pmatrix} 0 & E(x) \\ \overline{E(x)} & 0 \end{pmatrix} + \begin{pmatrix} 0 & \mu_1 \\ \mu_2 & 0 \end{pmatrix}\right)\begin{pmatrix}\psi_1(x)\\ \psi_2(x)\end{pmatrix}$$

$$= \begin{pmatrix} E(x)\psi_1(x) + \mu_1 \int\limits_{T^2} \psi_1(s)\,ds \\ \overline{E(x)}\psi_1(x) + \mu_2 \int\limits_{T^2} \psi_2(s)\,ds \end{pmatrix} = \begin{pmatrix} E(x)\psi_1(x) + \mu_1 C_1 \\ \overline{E(x)}\psi_1(x) + \mu_2 C_2 \end{pmatrix}$$

Bu tengliklarga (4) ni keltirib qo'yamiz:

$$H\psi = \begin{pmatrix} E(x)\left(-\frac{\mu_1 \overline{E(x)}}{|E(x)|^2 - z^2} C_1 - \frac{z\mu_2}{|E(x)|^2 - z^2} C_2\right) + \mu_1 C_1 \\ \overline{E(x)}\left(-\frac{\mu_2 E(x)}{|E(x)|^2 - z^2} C_2 - \frac{z\mu_1}{|E(x)|^2 - z^2} C_1\right) + \mu_2 C_2 \end{pmatrix} =$$

$$\begin{pmatrix} -\frac{\mu_1 |E(x)|^2}{|E(x)|^2 - z^2} C_1 + \mu_1 C_1 - \frac{z\mu_2 E(x)}{|E(x)|^2 - z^2} C_2 \\ -\frac{z\mu_1}{|E(x)|^2 - z^2} C_1 - \frac{\mu_2 |E(x)|^2}{|E(x)|^2 - z^2} C_2 + \mu_2 C_2 \end{pmatrix} =$$

$$\begin{pmatrix} \frac{-|E(x)|^2 + |E(x)|^2 - z^2}{|E(x)|^2 - z^2} \mu_1 C_1 - \frac{z\mu_2 E(x)}{|E(x)|^2 - z^2} C_2 \\ -\frac{z\mu_1}{|E(x)|^2 - z^2} C_1 + \frac{-|E(x)|^2 + |E(x)|^2 - z^2}{|E(x)|^2 - z^2} \mu_2 C_2 \end{pmatrix} =$$

$$\begin{pmatrix} \frac{-z^2}{|E(x)|^2 - z^2} \mu_1 C_1 - \frac{z\mu_2 E(x)}{|E(x)|^2 - z^2} C_2 \\ -\frac{z\mu_1}{|E(x)|^2 - z^2} C_1 - \frac{z^2}{|E(x)|^2 - z^2} \mu_2 C_2 \end{pmatrix} =$$

$$z\begin{pmatrix} \frac{-z}{|E(x)|^2 - z^2} \mu_1 C_1 - \frac{\mu_2 E(x)}{|E(x)|^2 - z^2} C_2 \\ -\frac{\mu_1}{|E(x)|^2 - z^2} C_1 - \frac{z}{|E(x)|^2 - z^2} \mu_2 C_2 \end{pmatrix} = z\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$$

yoki bundan

$$H\psi = z\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$$

tenglikka ega bo‘lamiz. Demak, $\psi = (\psi_1, \psi_2)$ H operatorning xos funksiyasi va z soni H operatorning xos qiymati ekan. Teorema isbot bo‘ldi.

Isbotlangan teoremaga asosan, aytish mumkinki H operatorning muhim spektri $[-1,1]$ dan iborat, $R\backslash[-1,1]$ da esa deskrit spektri mavjud.

Foydalanilgan adabiyotlar

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