DAVRIY FUNKSIYALAR SINFIDA KAUPNING YUKLANGAN HADLI SISTEMASINI INTEGRALLASH

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Annotatsiya: Ushbu maqolada yuklangan hadli nochiziqli differensial tenglamalarni davriy funksiyalar sinfida integrallash masalasi o'rganiladi. Tadqiqot obyekti sifatida Korteveg-de Friz, nochiziqli Shredinger va Kaup tenglamalari tanlangan. Tahlil davomida Floke yechimlari, Lyapunov funksiyasi, Dubrovin-Trubovits sistemasi, izlar formulalari hamda teskari spektral masala metodlari qo'llanilgan. Olingan natijalar orqali ushbu tenglamalarning davriy yechimlari aniqlangan bo‘lib, bu yondashuvlarning boshqa yuqori tartibli tenglamalarga nisbatan qo‘llanilishi taklif etiladi.

Kalit so‘zlar: Yuklangan hadli tenglama, davriy funksiyalar, Korteveg-de Friz tenglamasi, nochiziqli Shredinger tenglamasi, Kaup sistemasining yechimlari, teskari spektral masala, Lyapunov funksiyasi, Floke yechimlari, Dubrovin-Trubovits sistemasi, izlar formulasi.

Kaupning yuklangan hadli sistemasini

$$\begin{cases} p_t = -6pp_x - q_x + \gamma(t)\cdot p\big|_{x=0}\cdot p_x \\ q_t = p_{xxx} - 4qp_x - 2pq_x + \gamma(t)\cdot p\big|_{x=0}\cdot q_x \end{cases} \tag{1}$$

ushbu

$$p(x,t)\big|_{t=0} = p_0(x),\quad q(x,t)\big|_{t=0} = q_0(x) \tag{2}$$

boshlang`ich shartlar bilan birga $x$ bo`yicha π davrli

$$p(x+\pi,t)\equiv p(x,t),\ q(x+\pi,t)\equiv q(x,t) \tag{3}$$

hamda ushbu

$$p(x,t), q(x,t)\in C_x^3(t>0)\cap C_t^1(t>0)\cap C(t\ge 0) \tag{4}$$

silliqlik shartlarini qanoatlantiruvchi haqiqiy funksiyalar sinfida ko`rib chiqamiz. Bu yerda $\gamma(t)$ berilgan haqiqiy uzluksiz funksiya, $p_0(x), q_0(x)\in C^3(R)$ berilgan haqiqiy $\pi$ davrli funksiyalar bo`lib, $q_0(x)>0$.

Teorema 1. Agar $p(x,t)$ va $q(x,t)$ funksiyalar juftligi (1)-(4) masalaning yechimi bo`lsa, u holda koeffitsiyentlari $p(x+\tau,t)$ va $q(x+\tau,t)$ bo`lgan Shturm-Liuvill operatorlari kvadratik dastasining spektri τ va t parametrlarga bog`liq bo`lmaydi, $\xi_n(\tau,t)$, $n\in Z\setminus\{0\}$ spektral parametrlari esa quyidagi Dubrovin-Trubovits sistemasini qanoatlantiradi:

$$\frac{\partial\xi_n}{\partial t}=2(-1)^n\sigma_n(\tau,t)sign(n)\sqrt{(\xi_n-\lambda_{2n-1})(\lambda_{2n}-\xi_n)}\times$$
$$\times h_n(\xi)\times\{2p(\tau,t)+2\xi_n(\tau,t)-\gamma(t)p(0,t)\},\ n\in Z\setminus\{0\}. \qquad (5)$$

Bu yerda

$$h_n(\xi)=h_n(...,\xi_{-1},\xi_1,...)=\sqrt{(\xi_n-\lambda_{-1})(\xi_n-\lambda_0)\prod_{k\neq n,0}\frac{(\xi_n-\lambda_{2k-1})(\xi_n-\lambda_{2k})}{(\xi_n-\xi_k)^2}}.$$

Bunda $\sigma_n(\tau,t)=\pm1$, $n\in Z\setminus\{0\}$ ishoralar $\xi_n(\tau,t)$ spektral parametr $[\lambda_{2n-1},\lambda_{2n}]$ o`z lakunasining chetiga kelganida qarama-qarshi ishoraga o`zgaradi. Bundan tashqari ushbu

$$\xi_n(\tau,t)\big|_{t=0}=\xi_n^0(\tau),\ \ \sigma_n(\tau,t)\big|_{t=0}=\sigma_n^0(\tau),\quad n\in Z\setminus\{0\}$$

boshlang`ich shartlar ham bajariladi. Bu yerda $\xi_n^0(\tau),\sigma_n^0(\tau)$, $n\in Z\setminus\{0\}$ lar $p_0(x+\tau)$ va $q_0(x+\tau)$ koeffitsientlarga mos keluvchi spektral parametrlardir.

Isbot. Ushbu

$$-y''+q(x+\tau,t)y+2\lambda\,p(x+\tau,t)y-\lambda^2y=0 \qquad (6)$$

Shturm-Liuvill tenglamalarining kvadratik dastasi uchun qo`yilgan

$$y(0)=0,\ y(\pi)=0$$

Dirixle masalasining $\xi_n=\xi_n(\tau,t)$, $n\in Z\setminus\{0\}$ xos qiymatlariga mos keluvchi normallangan xos funksiyalarni $y_n(x,\tau,t)$, $n\in Z\setminus\{0\}$ orqali belgilaymiz.

Ushbu

$$-(y_n'',y_n)+(qy_n,y_n)+2\xi_n(py_n,y_n)-\xi_n^2=0$$

ayniyatni t bo`yicha differensiallab, quyidagi tenglikka ega bo`lamiz

$$-(\dot{y}_n'',y_n)-(y_n'',\dot{y}_n)+(q_ty_n+q\dot{y}_n,y_n)+(qy_n,\dot{y}_n)+$$
$$+2\dot{\xi}_n(py_n,y_n)+2\xi_n(p_ty_n+p\dot{y}_n,y_n)+2\xi_n(py_n,\dot{y}_n)-2\xi_n\dot{\xi}_n=0. \qquad (7)$$

Bu yerda $L_2(0,\pi)$ fazoning skalyar ko`paytmasi ishlatildi.

Oxirgi tenglikni quyidagi tarzda yozib olamiz

$$\left(-\dot{y}_n''+q\dot{y}_n+2\xi_np\dot{y}_n,y_n\right)+\left(-y_n''+qy_n+2\xi_npy_n,\dot{y}_n\right)+$$
$$+\left(q_ty_n+2\xi_np_ty_n,y_n\right)+2\dot{\xi}_n(py_n,y_n)-2\xi_n\dot{\xi}_n=0,$$
$$2\dot{\xi}_n[\xi_n-(py_n,y_n)]=\left(q_ty_n+2\xi_np_ty_n,y_n\right),$$

ya'ni

$$2\dot{\xi}_n\left(\xi_n-\int_0^\pi py_n^2dx\right)=\int_0^\pi(q_t+2\xi_np_t)y_n^2dx. \qquad (8)$$

Ushbu

$$p_t(x+\tau,t)=-6p(x+\tau,t)p_x(x+\tau,t)-q_x(x+\tau,t)+\gamma(t)p(0,t)p_x(x+\tau,t),$$
$$q_t(x+\tau,t)=p_{xxx}(x+\tau,t)-4q(x+\tau,t)p_x(x+\tau,t)-$$
$$-2p(x+\tau,t)q_x(x+\tau,t)+\gamma(t)p(0,t)q_x(x+\tau,t)$$

ayniyatlardan foydalanib, (8) tenglikni quyidagi tarzda yozib olamiz

$$2\dot{\xi}_n\left(\xi_n-\int_0^\pi py_n^2dx\right)=\int_0^\pi\{p_{xxx}-4qp_x-2pq_x+\gamma(t)p(0,t)q_x+$$
$$+2\xi_n[-6pp_x-q_x+\gamma(t)p(0,t)p_x]\}y_n^2dx. \qquad (9)$$

Integral ostidagi funksiyaning boshlang`ichini $y_n$ va $y_n'$ ga nisbatan kvadratik forma ko`rinishida izlaymiz, ya'ni

$$\{ay_n^2+by_ny_n'+cy_n'^2\}'=$$
$$=\{p_{xxx}-4qp_x-2pq_x+\gamma(t)p(0,t)q_x+2\xi_n[-6pp_x-q_x+\gamma(t)p(0,t)p_x]\}y_n^2. \quad (10)$$

Bu yerda $a=a(x,\tau,t,\xi_n)$, $b=b(x,\tau,t,\xi_n)$, $c=c(x,\tau,t,\xi_n)$ lar y_n va y_n' ga bog`liq emas. (10) tenglik chap tomonidagi hosilalarni hisoblab, ushbu

$$y_n''=[q+2p\xi_n-\xi_n^2]y_n$$

ayniyatlardan foydalansak, quyidagi tenglik hosil bo`ladi

$$(a'+bq+2bp\xi_n-b\xi_n^2)y_n^2+(2a+b'+2cq+4pc\xi_n-2c\xi_n^2)y_ny_n'+(b+c')y_n'^2=$$
$$=\{p_{xxx}-4qp_x-2pq_x+\gamma(t)p(0,t)q_x+2\xi_n[-6pp_x-q_x+\gamma(t)p(0,t)p_x]\}y_n^2.$$
(11)

Bunga ko`ra

$$b=-c',\quad a=\frac{1}{2}c''+c\cdot(\xi_n^2-2p\xi_n-q),$$
$$p_{xxx}-4qp_x-2pq_x+\gamma(t)p(0,t)q_x+2\xi_n[-6pp_x-q_x+\gamma(t)p(0,t)p_x]=$$
$$=\frac{1}{2}c'''+2c'\cdot(\xi_n^2-2p\xi_n-q)-c\cdot(2p'\xi_n+q'). \qquad (12)$$

Oxirgi tenglikning chap tomoni ξ_n ning chiziqli funksiyasi bo`lgani uchun o`ng tomoni ham ξ_n ning chiziqli funksiyasi bo`lishi kerak. $c(x,\tau,t,\xi_n)$ ni $\xi_n$ ga nisbatan 1-darajali ko`phad ko`rinishida izlaymiz:

$$c(x,\tau,t,\xi_n)=c_0(x,\tau,t)\xi_n+c_1(x,\tau,t). \qquad (13)$$

(13) ifodani (12) tenglikka qo`ysak va ξ_n ning mos darajalari oldidagi koeffitsientlarni taqqoslasak, ushbu

$$c_0(x,\tau,t)=2,\quad c_1(x,\tau,t)=2p(x+\tau,t)-\gamma(t)p(0,t) \qquad (14)$$

tengliklarga ega bo`lamiz.

(10) ayniyatga ko`ra

$$2\dot{\xi}_n\left(\xi_n-\int_0^{\pi}py_n^2dx\right)=\left\{[\frac{1}{2}c''+c\cdot(\xi_n^2-2p\xi_n-q)]y_n^2-c'y_ny_n'+cy_n'^2\right\}\Big|_0^{\pi}=$$

$$=c(\pi,\tau,t,\xi_n)y_n'^2(\pi,\tau,t)-c(0,\tau,t,\xi_n)y_n'^2(0,\tau,t)\,. \qquad (15)$$

Ushbu $c(x,\tau,t,\xi_n)$ funksiya x bo\`yicha $\pi$ davrli ekanini hisobga olsak, (15) tenglik quyidagi ko\`rinishni oladi

$$2\dot{\xi}_n\left(\xi_n-\int_0^{\pi}py_n^2dx\right)=c(0,\tau,t,\xi_n)[y_n'^2(\pi,\tau,t)-y_n'^2(0,\tau,t)]. \qquad (16)$$

Bu yerda ushbu

$$c(x,\tau,t,\xi_n)=2\xi_n+2p(x+\tau,t)-\gamma(t)p(0,t)$$

ifodadan foydalansak, quyidagi

$$2\dot{\xi}_n\left(\xi_n-\int_0^{\pi}py_n^2dx\right)=\{2\xi_n+2p(\tau,t)-\gamma(t)p(0,t)\}[y_n'^2(\pi,\tau,t)-y_n'^2(0,\tau,t)] \qquad (17)$$

tenglik kelib chiqadi.

$$2\int_0^{\pi}[\lambda-p(x+\tau,t)]s^2(x,\lambda,\tau,t)dx=s'(\pi,\lambda,\tau,t)\frac{\partial s(\pi,\lambda,\tau,t)}{\partial\lambda}-s(\pi,\lambda,\tau,t)\frac{\partial s'(\pi,\lambda,\tau,t)}{\partial\lambda}$$

formuladan foydalansak, quyidagi tenglikni olamiz:

$$2\xi_n\alpha_n^2-2\int_0^{\pi}p(x+\tau,t)s^2(x,\xi_n,\tau,t)dx=s'(\pi,\xi_n,\tau,t)\frac{\partial s(\pi,\xi_n,\tau,t)}{\partial\lambda}. \qquad (18)$$

Bu yerda

$$\alpha_n^2=\int_0^{\pi}s^2(x,\xi_n(t),\tau,t)dx\,.$$

Ushbu

$$y_n(x,\tau,t)=\frac{1}{\alpha_n}s(x,\xi_n(t),\tau,t)$$

ifodani (17) formulaga qo\`yib, (18) tenglikdan foydalanamiz:

$$2\dot{\xi}_n\left(\xi_n\alpha_n^2-\int_0^{\pi}ps^2(x,\xi_n,\tau,t)dx\right)=$$

$$=\{2\xi_n+2p(\tau,t)-\gamma(t)p(0,t)\}\cdot[s'^2(\pi,\xi_n,\tau,t)-1],$$

$$\dot{\xi}_ns'(\pi,\xi_n,\tau,t)\frac{\partial s(\pi,\xi_n,\tau,t)}{\partial\lambda}=\{2\xi_n+2p(\tau,t)-\gamma(t)p(0,t)\}\cdot[s'^2(\pi,\xi_n,\tau,t)-1],$$

$$\dot{\xi}_n\frac{\partial s(\pi,\xi_n,\tau,t)}{\partial\lambda}=\{2\xi_n+2p(\tau,t)-\gamma(t)p(0,t)\}\cdot\left(s'(\pi,\xi_n,\tau,t)-\frac{1}{s'(x,\xi_n,\tau,t)}\right). \qquad (19)$$

Ushbu

$$c(x,\lambda,\tau,t)s'(x,\lambda,\tau,t)-c'(x,\lambda,\tau,t)s(x,\lambda,\tau,t)=1$$

Vronskiy ayniyatida $x=\pi$ va $\lambda=\xi_n$ desak,

$$c(\pi,\xi_n,\tau,t)=\frac{1}{s'(\pi,\xi_n,\tau,t)} \qquad (20)$$

kelib chiqadi. Bu tenglikdan hamda ushbu

$$[c(\pi,\lambda,\tau,t)-s'(\pi,\lambda,\tau,t)]^2=(\Delta^2(\lambda)-4)-4c'(\pi,\lambda,\tau,t)s(\pi,\lambda,\tau,t)$$

ayniyantdan foydalanib, quyidagini hosil qilamiz

$$s'(\pi,\xi_n,\tau,t)-\frac{1}{s'(\pi,\xi_n,\tau,t)}=\sigma_n(\tau,t)\sqrt{\Delta^2(\xi_n)-4}\,. \qquad (21)$$

Bu yerda

$$\Delta(\lambda)=c(\pi,\lambda,t)+s'(\pi,\lambda,t),\ \sigma_n(\tau,t)=sign\left\{s'(\pi,\xi_n,\tau,t)-\frac{1}{s'(\pi,\xi_n,\tau,t)}\right\}.$$

Agar (21) ifodani (19) ga qo`ysak, quyidagi tenglikni olamiz

$$\dot{\xi}_n=\{2\xi_n+2p(\tau,t)-\gamma(t)p(0,t)\}\cdot\frac{\sigma_n(\tau,t)\sqrt{\Delta^2(\xi_n)-4}}{\dfrac{\partial s(\pi,\xi_n,\tau,t)}{\partial\lambda}}. \qquad (22)$$

Ushbu

$$\Delta^2(\lambda)-4=-4\pi^2(\lambda-\lambda_{-1})(\lambda-\lambda_0)\prod_{0\neq k=-\infty}^{\infty}\frac{(\lambda-\lambda_{2k-1})(\lambda-\lambda_{2k})}{k^2},$$

$$s(\pi,\lambda,\tau,t)=\pi\prod_{0\neq k=-\infty}^{\infty}\frac{\xi_k-\lambda}{k}$$

yoyilmalardan foydalanib, (22) ayniyatni quyidagi tarzda yozamiz:

$$\dot{\xi}_n=2(-1)^n\sigma_n(\tau,t)sign(n)\cdot\sqrt{(\xi_n-\lambda_{2n-1})(\lambda_{2n}-\xi_n)}\times h_n(\xi)\times$$
$$\times\{2\xi_n+2p(\tau,t)-\gamma(t)p(0,t)\}. \qquad (23)$$

Bunda biz quyidagi tenglikdan ham foydalandik:

$$sign\left\{-\frac{\pi}{n}\prod_{k\neq n,0}\frac{\xi_k-\xi_n}{k}\right\}=(-1)^n sign(n)\,.$$

Demak, (5) tenglik kelib chiqdi.

Agar chegaraviy shartlarni davriy yoki antidavriy shartlar bilan almashtirsak, (17) tenglamalar o`rnida $\dot{\lambda}_n=0$, $n\in Z$ tenglamalar hosil bo`ladi. Demak, λ_n, $n\in Z$ davriy va antidavriy masalaning xos qiymatlari t parametrga bog`liq emas ekan. **Teorema 1 isbotlandi.**

Izox 1. Ushbu izlar formulasi

$$p(\tau,t)=\frac{\lambda_{-1}+\lambda_0}{2}+\sum_{0\neq k=-\infty}^{\infty}\left(\frac{\lambda_{2k-1}+\lambda_{2k}}{2}-\xi_k(\tau,t)\right) \qquad (24)$$

yordamida (5) sistemani “yopiq” ko`rinishda yozish mumkin.

Natija 1. Yuqoridagi 1-teorema (1)-(4) masalani yechish usulini beradi:

1) Avvalo $p_0(x+\tau)$ va $q_0(x+\tau)$ koeffitsientli Shturm-Liuvill tenglamalarining kvadratik dastasi uchun λ_n, $n\in Z$, $\xi_n^0(\tau)$, $\sigma_n^0(\tau)$, $n\in Z\setminus\{0\}$ spektral berilganlarini topamiz;

2) So`ngra, (5)+(6) Koshi masalasini $\tau=0$ bo`lganida yechib, $\xi_n(0,t)$, $n\in Z\setminus\{0\}$ spektral parametrlarni topamiz hamda (24) formula yordamida $p(0,t)$ ni aniqlaymiz;

3) Shundan so`ng, (5)+(6) Koshi masalasini τ parametrning ixtiyoriy qiymatida yechib, $\xi_n(\tau,t)$, $\sigma_n(\tau,t)$, $n\in Z\setminus\{0\}$ spektral parametrlarni topamiz;

4) Bu yechimlarni (24) va quyidagi

$$q(\tau,t)+2p^2(\tau,t)=\frac{(\lambda_{-1})^2+(\lambda_0)^2}{2}+\sum_{0\neq k=-\infty}^{\infty}\left(\frac{(\lambda_{2k-1})^2+(\lambda_{2k})^2}{2}-\xi_k^2(\tau,t)\right)$$

izlar formulasiga qo`yib, $p(x,t)$ va $q(x,t)$ funksiyalarni aniqlaymiz.

Izox 2. Yuqoridagi usul yordamida tuzilgan $p(\tau,t)$, $q(\tau,t)$ funksiyalar (1) sistemani qanoatlantirishini ko`rsatamiz. Buning uchun Dubrovinning quyidagi sistemasidan

$$\frac{\partial\xi_n}{\partial\tau}=2(-1)^{n-1}sign(n)\sigma_n(\tau,t)h_n(\xi)\sqrt{(\xi_n-\lambda_{2n-1})(\lambda_{2n}-\xi_n)},\ n\in Z\setminus\{0\} \qquad (25)$$

va ushbu

$$-\frac{3}{4}p_{\tau\tau}(\tau,t)+4p^3(\tau,t)+3p(\tau,t)q(\tau,t)=$$

$$=\frac{(\lambda_{-1})^3+(\lambda_0)^3}{2}+\sum_{0\neq k=-\infty}^{\infty}\left(\frac{(\lambda_{2k-1})^3+(\lambda_{2k})^3}{2}-\xi_k^3(\tau,t)\right) \qquad (26)$$

izlar formulasidan ham foydalanamiz ([8]). (5) va (9) sistemalarga ko`ra

$$\frac{\partial\xi_k}{\partial t}=-\{2p+2\xi_k-\gamma(t)p(0,t)\}\frac{\partial\xi_k}{\partial\tau},\ k\in Z\setminus\{0\}. \qquad (27)$$

Agar (7) izlar formulasini t bo`yicha differensiallab, (11) ayniyatlarni e'tiborga olsak, ushbu

$$p_t=-\sum_{0\neq k=-\infty}^{\infty}\frac{\partial\xi_k}{\partial t}=2p\sum_{0\neq k=-\infty}^{\infty}\frac{\partial\xi_k}{\partial\tau}+2\sum_{0\neq k=-\infty}^{\infty}\xi_k\frac{\partial\xi_k}{\partial\tau}-\gamma(t)p(0,t)\sum_{0\neq k=-\infty}^{\infty}\frac{\partial\xi_k}{\partial\tau} \qquad (28)$$

tenglik kelib chiqadi. (7) va (8) izlar formulalaridan τ bo`yicha hosila olamiz:

$$\sum_{0\neq k=-\infty}^{\infty}\frac{\partial\xi_k}{\partial\tau}=-p_\tau,\quad 2\sum_{0\neq k=-\infty}^{\infty}\xi_k\frac{\partial\xi_k}{\partial\tau}=-4pp_\tau-q_\tau. \qquad (29)$$

Bu ifodalarni (28) tenglikka qo`yib, ushbu

$$p_t=-6pp_\tau-q_\tau+\gamma(t)p(0,t)p_\tau \qquad (30)$$

ayniyatni olamiz.

Endi (24) izlar formulasini t bo`yicha differensiallaymiz va (27) ayniyatlarni ishlatamiz

$$q_t = -4pp_t - 2\sum_{0\neq k=-\infty}^{\infty} \xi_k \frac{\partial \xi_k}{\partial t} =$$

$$= -4pp_t + 4p\sum_{0\neq k=-\infty}^{\infty} \xi_k \frac{\partial \xi_k}{\partial \tau} + 4\sum_{0\neq k=-\infty}^{\infty} \xi_k^2 \frac{\partial \xi_k}{\partial \tau} - 2\gamma(t)p(0,t)\sum_{0\neq k=-\infty}^{\infty} \xi_k \frac{\partial \xi_k}{\partial \tau}.$$

Bu tenglikka (29) ifodalarni qo`ysak, hamda (26) izlar formulasidan foydalansak, ushbu

$$q_t = -4pp_t + 2p(-4pp_\tau - q_\tau) + (p_{\tau\tau\tau} - 16p^2 p_\tau - 4p_\tau q - 4pq_\tau) -$$
$$-\gamma(t)p(0,t)(-4pp_\tau - q_\tau)$$

ayniyat kelib chiqadi. Bu yerga (30) ifodani qo`ysak, u quyidagi ko`rinishni oladi

$$q_t = p_{\tau\tau\tau} - 4qp_\tau - 2pq_\tau + \gamma(t)p(0,t)q_\tau.$$

Demak, tuzilgan $p(\tau,t)$, $q(\tau,t)$ funksiyalar (1) sistemani qanoatlantirar ekan.

Natija 2. Agar boshlang`ich shartlardagi $p_0(x)$ va $q_0(x)$ funksiyalar haqiqiy analitik funksiya bo`lsa, u holda unga mos keluvchi lakunalar uzunliklari eksponensial ravishda nolga intiladi, bu lakunalar $p(x,t)$ va $q(x,t)$ funksiylaraga ham mos keladi. Shuning uchun $p(x,t)$ va $q(x,t)$ yechimlar x o`zgaruvchi bo`yicha haqiqiy analitik funksiya bo`ladi ([12]).

Natija 3. Agar boshlang`ich shartlardagi $p_0(x)$ va $q_0(x)$ funksiyalar $\frac{\pi}{2}$ davrga ega bo`lsa, u holda unga mos keluvchi barcha toq nomerli lakunalar yo`qoladi, bu lakunalar $p(x,t)$ va $q(x,t)$ koefitsiyentlarga ham mos keladi. Shuning uchun $p(x,t)$ va $q(x,t)$ yechimlar $x$ o`zgaruvchi bo`yicha $\frac{\pi}{2}$ davrga ega bo`ladi ([13]).

Foydalanilgan adabiyotlar.

1. Ахмедиев Н.Н., Корнеев В.И. Модуляционная неустойчивость и периодические решения нелинейного уравнения Шредингера. // ТМФ, 1986, т. 69, № 2, с. 189-194
2. Ахмедиев Н.Н., Елеонский В.М., Кулагин Н.Е. Точные решения первого порядка нелинейного уравнения Шредингера. // ТМФ, 72, № 2, 1987, с. 183-196.
3. Alfimov G.L., Its A.R., Kulagin N.E. Modulation instability of solutions of the nonlinear Schrödinger equation. //Theoret. Mat. Fiz., 84:2, 163-172 (1990).
4. Alisher Yakhshimuratov. The Nonlinear Schrodinger Equation with a Self-consistent Source in the Class of Periodic Functions. // Mathematical Physics, Analysis and Geometry, (2011) 14, pp.153-169.

5. А. О. Смирнов, Вещественные конечнозонные регулярные решения уравнения Каупа-Буссинеска. *Теорет. мат.физ.*, **66**:1, (1986), 30-46.
6. А. О. Смирнов, Матричный аналог теоремы Аппеля и редукции многомерных тэта-функций Римана. *Мат. сб.*, **133**(**175**):3(7), (1987), 382-391.
7. А. Б. Хасанов, А. Б. Яхшимуратов, Об уравнении Кортевега-де Фриза с самосогласованным источником в классе периодических функций. *Теорет. мат.физ.*, 164:2, 214-221.
8. Alisher Yakhshimuratov. The Nonlinear Schrödinger Equation with a Self-consistent Source in the Class of Periodic Functions. // Mathematical Physics, Analysis and Geometry, (2011) 14, pp.153-169, DOI 10.1007/s11040-011-9091-5.
9. А. Б. Яхшимуратов, Аналог обратной теоремы Борга для квадратичного пучка операторов Штурма-Лиувилля. *Вестник Елецкого государственного университета им. И.А.Бунина, серия «Математика. Компьютерная математика»*, **8**:1, (2005), 121-126.
10. Б. А. Бабажанов, А. Б. Хасанов, А. Б. Яхшимуратов, Об обратной задаче для квадратичного пучка операторов Штурма-Лиувилля с периодическим потенциалом. *Дифференциальные уравнения*, **41**:3, (2005), 298-305.