A GENERALIZED DIRECT METHODS FOR THE LOADED NONLINEAR DEGASPERIS-PROCESI EQUATION

M.M.Xasanov*, Sh.Sh.Omonov**
**Urgench State University, Urgench, Uzbekistan*
***Tashkent State University of Economic, Tashkent, Uzbekistan*
e-mail: omonov.sherzod.4808@mail.ru

Abstract: This paper is studied to finding the traveling wave solutions of the loaded nonlinear Degasperis-Procesi equation. By using the dynamical system theory the nonlinear Degasperis-Processi equation are studied .The bounded travelling wave solutions such as peakons are analytically described. The loaded Degasperis-Procesi equation is converted to the ordinary differential equation which are solved for all possible soliton solutions of Degasperis-Procesi equation. We construct exact travelling wave solution for loaded nonlinear Degasperis-Procesi equation,and the obtained solution agrees well with the previously known result.

Keywords: Loaded nonlinear Degasperis-Procesi equation,Travelling wave solution. AMS Subject Classification 2010:

1. Introduction.

Degasperis and Procesi [1]showed, by the use of the method of asymptotic integrability, that the PDE

$$u_t - u_{xxt} + (b+1)uu_x = bu_x u_{xx} + uu_{xxx} \tag{1}$$

cannot be completely integrable unless $b=2$ or $b=3$ .The case $b=2$ is the following Camassa-Holm (CH) shallow water equation (see [2])

$$u_t - u_{xxt} + 3uu_x = 2u_x u_{xx} + uu_x \tag{2}$$

which is well known to be integrable and topossess multi-peakon solutions. The case $b=3$ is the following Degasperis-Procesi (DP) shallow water equation

$$u_t - u_{xxt} + 4uu_x = 3u_x u_{xx} + uu_{xxx} \tag{3}$$

Although, the DP equation (3) has a similar form to the Ch equation (2), two equations are pretty different. For two equations, the different isospectral problem and the fact that there is no simple transformation of equation (3) into equation (2) imply that equation (3) is different from equation (2) in the integrable structures and the form of the conservation laws. The DP equation (3) is very interesting as it is an integrable shallow water equation and presents a quite rich structure. Degasperis, Holm and Hone [3-5]proved that equation (3) is integrable by constructing its lax pair, and admits multi-peakon solutions, and explained connection with a negative flow in the Kaup-kupershmidt hierarchy via a neciprocal transformation. Landmark and Szmigielski [6] presented an inverse scattering approach for computing n-peakon solutions of the equation (3). The blow-upphenomenon of equation (3) was discussed and the global existence of the solution was proved in [7]. In [8,9] the Cauchy problem for equation (3) was demonstrated. Much work on the DP equation (3) has been done [10-11].

It is well known that nonlinear phenomena exist everywhere. For example, they exist influid physics, condensed matter physics ,biophysics, plasma physics, quantum field theory ,particle physics and nonlinear optics etc. they also connect with our everyday's life. Generally speaking, nonlinear phenomena can be described by nonlinear partial differential equations .

It is known that loaded differential equations have great practical applications. In the literature [12-16], loaded differential equations are typically called equations containing in the coefficients or in the right-hand side any functionals of the solution, in particular, the values of the solution or its derivatives on many folds of lower dimension. These types of equations were explored in the works of N.N. Nazarov and N.N. Kochin. However, they did not use the term "loaded equation". At first, the term has been used in the works of A.M. Nakhushev, which the most general definition of a loaded equation is given and various loaded equations are classified in details[16]. For instance, loaded differential, integral, integro-differential, functional equations etc., and numerous applications are described.

Alternatively, the (G'/G) - expansion method [17-28] is also effective in finding traveling wave solutions of nonlinear evolution equations. Integration of the loaded modified Korteweg-de Vries (mKdV) equation in the class of periodic functions is studied in [29]. (G'/G)-expansion method wasused for the integrations of loaded Korteweg-de Vries (KdV) equation and the loaded modified Korteweg-de Vries (mKdV) equation in [30,31].

In this paper, the solution of the loaded nonlinear Degasperis-Procesi equation is studied by usage of direct method.

Let us consider the following loaded nonlinear Degasperis-Procesi equation

$$u_t - u_{xxt} + 4uu_x = 3u_x u_{xx} + uu_{xxx} + \gamma(t)u(0,t)(4uu_x - cu_x) \quad (4)$$

where $u(x,t)$ is an unknown function, $x \in R$, $t \ge 0, \gamma(t)$-is the given real continuous function.

2.The loaded nonlinear Degasperis-Procesi Equation

Assume the solution of loaded nonlinear Degasperis - Procesi equation as

$$u = \varphi(x - ct),\ \xi = x - ct,\ u = \varphi(\xi)$$

Now, we determine partial derivatives $u = \varphi(x - ct)$, derivative with respect to variables x and t .

$$u_t(x,t) = -c\varphi',\ u_x(x,t) = \varphi',\ u_{xx}(x,t) = \varphi'',\ u_{xxx}(x,t) = \varphi''',\ u_{xxt}(x,t) = -c\varphi''', \quad (5)$$

Then, by using above equation (4) is transformed into the following ordinary differential equation

$$-c\varphi' + c\varphi''' + 4\varphi\varphi' - 3\varphi'\varphi'' - \varphi\varphi''' - \gamma(t)u(0,t)(4\varphi\varphi' - c\varphi') = 0. \qquad (6)$$

By one time integrating with respect to ξ, equation (6) becomes

$$-c\varphi + c\varphi'' + 2\varphi^2 - \varphi'^2 - \varphi\varphi'' - \gamma(t)u(0,t)(2\varphi^2 - c\varphi) = 0,$$

$$(c-\varphi)\varphi'' = \varphi'^2 - (2\varphi - c)\varphi + \gamma(t)u(0,t)(2\varphi^2 - c\varphi). \qquad (7)$$

Then we get the following substitutions

$$\varphi' = p,\ \ \varphi'' = pp',\ p = p(\varphi),$$

equation (7) using substitutions made following visible

$$(c-\varphi)\,pp' = p^2 - (2\varphi - c)\varphi + \gamma(t)u(0,t)(2\varphi^2 - c\varphi),$$

and also by using following substitution

$$p^2 = z,\ \ pp' = \frac{z'}{2},$$

we take the first order linear differential equation

$$(c-\varphi)\frac{z'}{2} = z^2 - (2\varphi - c)\varphi + \gamma(t)u(0,t)(2\varphi^2 - c\varphi),$$

$$z' - \frac{2}{(c-\varphi)}z = -2\frac{(2\varphi - c)\varphi}{(c-\varphi)} + 2\frac{\gamma(t)u(0,t)(2\varphi^2 - c\varphi)}{(c-\varphi)}, \qquad (8)$$

and to find its solution, we perform the following calculations:

$$z' - \frac{2}{(c-\varphi)}z = 2\frac{(\gamma(t)u(0,t)-1)(2\varphi^2 - c\varphi)}{(c-\varphi)}$$

$$z = \frac{1}{(c-\varphi)^2}\left[k + 2(\gamma(t)u(0,t)-1)\int (c-\varphi)(2\varphi^2 - c\varphi)d\varphi\right]$$

$$z = \frac{1}{(c-\varphi)^2}\left[k + 2(\gamma(t)u(0,t)-1)\int (3c\varphi^2 - c^2\varphi - 2\varphi^3)d\varphi\right]$$

$$z = \frac{1}{(c-\varphi)^2}\left[k + 2(\gamma(t)u(0,t)-1)(\frac{3c\varphi^3}{3} - \frac{c^2\varphi^2}{2} - \frac{2\varphi^4}{4})\right]$$

if $k = 0$, z function becomes like this.

$$z = -\frac{(\gamma(t)u(0,t)-1)(c^2\varphi^2 - 2c\varphi^3 + \varphi^4)}{(c-\varphi)^2},$$

$$z = \frac{\varphi^2(1-\gamma(t)u(0,t))(c^2 - 2c\varphi + \varphi^2)}{(c^2 - 2c\varphi + \varphi^2)},$$

$$z = \varphi^2(1-\gamma(t)u(0,t)),\ \ p^2 = \varphi^2(1-\gamma(t)u(0,t)),$$

$$p = \pm\varphi\sqrt{1-\gamma(t)u(0,t)},$$

$$\frac{d\varphi}{d\xi} = \pm\varphi\sqrt{1-\gamma(t)u(0,t)},$$

$$\frac{d\varphi}{\varphi} = \sqrt{1-\gamma(t)u(0,t)}dx - c\sqrt{1-\gamma(t)u(0,t)}dt.$$

Now we find φ by integrating both sides of the equation:

$$\int\frac{d\varphi}{\varphi} = \sqrt{1-\gamma(t)u(0,t)}x - c\int_0^t\sqrt{1-\gamma(s)u(0,s)}ds,$$

$$\ln\varphi = \sqrt{1-\gamma(t)u(0,t)}x - c\int_0^t\sqrt{1-\gamma(s)u(0,s)}ds + C,$$

$$\varphi = Ce^{\sqrt{1-\gamma(t)u(0,t)}x - c\int_0^t\sqrt{1-\gamma(s)u(0,s)}ds}.$$

Substituting equation φ into $u(x,t)$,we obtain the solution of $u(x,t)$

$$u(x,t) = e^{\sqrt{1-\gamma(t)u(0,t)}x - c\int_0^t\sqrt{1-\gamma(s)u(0,s)}ds}. \quad (9)$$

Result.1) from $u(x,t)$ on $x=0$ ni (9) we find the function $u(0,t)$.

2) Substituting the last function $u(0,t)$ into formula (9), we get the solution of loaded nonlinear Degasperis-Procesi equation.

REFERENCES

1. Escher, Joachim, Liu Yue, Yin Z Y. Shock waves and blow-up phenomena for the periodic Degasperis–Procesi equation. *Indiana Univ. Math. J.*,56 (2007): 87–117.
2. Baltaeva U., Torres P.J. Analog of the Gellerstedt problem for a loaded equation of the third order. Math. Meth. Appl. Sci., 2019, vol. 42, p. 3865.
3. Cannon J.R., Yin H.M. On a class of nonlinear nonclassical parabolic problems. J. Different. Equat., 1989, vol. 79, pp. 266–288. https://doi.org/10.1080/00036810500277876
4. Chadam J.M., Peirce A., Yin H.M. The blowup property of solutions to some diffusion equations with localized nonlinear reactions. J. Math. Analys. and Appl., 1992, vol. 169, p. 313.
5. Kadomtsev B.B., Karpman V.I. Nonlinear waves. Sov. Phys. Usp., 1971, vol. 14, p. 40.
6. Nakhushev A.M. On nonlocal problems with shift and their connection with loaded equations. Differents. Uravn., 1985, vol. 21, p. 92.
7. Bekir A. Application of the (G'/G)-expansion method for nonlinear evolution equations. Phys. Lett. A, 2008, vol. 372, p. 3400. http://dx.doi.org/10.1016/j.physleta.2008.01.057

8. Bekir A., Guner O. Exact solutions of nonlinear fractional differential equations by (G'/G)-expansion method. Chin. Phys. B, 2013, vol. 22, p. 110202. https://doi.org/10.1088/1674-1056/22/11/110202
9. Li Z.L. Constructing of new exact solutions to the GKdV-mKdV equation with any-order nonlinear terms by (G'/G)-expansion method. Appl. Math. Comput., 2010, vol. 217, p. 1398. https://doi.org/10.1016/j.amc.2009.05.034
10. Shang N., Zheng B. Exact Solutions for Three Fractional Partial Differential Equations by the (G'/G) Method. Int. J. Appl. Math., 2013, vol. 43, p. 114.
11. University. Series Mathematics, 2022, vol. 41, pp. 85–95. https://doi.org/10.26516/1997-7670.2022.41.85