RATSIONAL VA IRRATSIONAL IFODALARNING INTEGRALLARINI HISOBLASHNING BA’ZI INNOVATSION USULLARI

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Annotatsiya. Ma’lumki noaniq va aniq integrallar matematik analizning katta qismini tashkil etadi.Bunda ulani hisoblash usullari va boshqa fanlarga oid masala-larni yechishga tadbiqlari qaraladi.Bunda ularga oid masalalarni yechishda talaba-lar tez - tez uslubiy qiyinchiliklarda duch keladilar.Shuning uchun bu maqolada bir qancha innovatsion usullar tavsiya etiladi.

Kalit so’zlar. Noananaviy noma’lun koefisentlar usuli,trigonometrik o’zgaruvchi almashtirishusuli ,geometrik usul.

ИННОВАЦИОННЫЕ МЕТОДЫ ВЫЧИСЛЕНИЯ ИНТЕГРАЛОВ НЕКОТОРЫХ РАЦИОНАЛЬНЫХ И ИРРАЦИОНАЛЬНЫХ ВЫРАЖЕНИЙ

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Аннотция. Как известно, неопределенные и точные интегралы составляют большую часть математического анализа.При этом будут рассмотрены методы расчета и их применение для решения задач, связанных с другими науками.При этом студенты часто сталкиваются с методическими трудностями при решении вопросов, связанных с ними.Вот почему в этой статье рекомендуется несколько инновационных методов.

Ключевые слова. Нетрадиционный метод коэффициентов, тригономет-рический метод подстановки переменных, геометрический метод.

INNOVATIVE METHODS FOR CALCULATING INTEGRALS OF SOME RATIONAL AND IRRATIONAL EXPRESSIONS

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Annotation. It is known that indefinite and definite integrals form a large part of mathematical analysis.In this case, the methods of calculating the connection and

their application to solving issues related to other disciplines are considered.In solving issues related to them in this, students are often faced with methodological difficulties.Therefore, several innovative methods are recommended in this article.

Keyvords. Non-traditional method of coefficients, trigonometric method of substitution of variables, geometric method.

O'quv jaroyonini samaradorligini oshirishda texnologik innovatsiyalardan foydalanish muammosi zamonaviy talablardan hisoblanadi.Shuning uchun oliy matematikani o'rganishda ana shu muammoni hal qilish uchun quyida bir nechta tavsiyalar keltiriladi.Ushbu muammoni muallif tomonidan o'rganishda [1,2,5,9] ishlarda integrative texnologiya , [3,4,6,7] ishlarda multi variant texnologiya , [8,10] ishlarda noananaviy interaktiv texnologiyadan foydalanish bayon etilgan.

1-**Misol.** Integralni hisoblang $I=\int\frac{1}{1-x+x^2}dx$

Yechish. **1-usul**.Agar biz $x=\frac{1+t}{2}$ deb belgilasak $dx=\frac{1}{2}dt$ bo'ladi. U holda bu integralni quyidagicha yozish mumkin

$$I=\int\frac{1}{1-x(1-x)}dx=\frac{1}{2}\int\frac{1}{1-\left(\frac{1+t}{2}\right)(1-\frac{1+t}{2})}dt=\frac{1}{2}\int\frac{1}{1-\left(\frac{1+t}{2}\right)(\frac{1-t}{2})}dt=\frac{1}{2}\int\frac{1}{\left(1-\frac{1-t^2}{4}\right)}dt=$$

$$=2\int\frac{1}{3+t^2}dt=2\int\frac{1}{\left(\sqrt{3}\right)^2+t^2}dt=\frac{2}{\sqrt{3}}arctg\left(\frac{t}{\sqrt{3}}\right)+C=\frac{2}{\sqrt{3}}arctg\left(\frac{2x-1}{\sqrt{3}}\right)+C.$$

2-usul.Agar biz $x=\frac{1+t}{2}$ desak $dx=\frac{1}{2}dt$ bo'ladi. Shuning uchun integral

$$I=\int\frac{1}{1-x+x^2}dx=4\int\frac{1}{4-4x+4x^2}dx=2\int\frac{1}{\left(2x-1^2\right)+3}d\left(2x\right)=2\int\frac{1}{\left(2x-1^2\right)+\left(\sqrt{3}\right)^2}d\left(2x\right)=$$

$$=\frac{2}{\sqrt{3}}arctg\frac{2x-1}{\sqrt{3}}+C$$

ga teng bo'ladi.Ma'lumki biz

$$\int\frac{px+q}{ax^2+bx+c}dx \text{ va } \int\frac{px+q}{\sqrt{ax^2+bx+c}}dx$$

ko'rinishdagi integrallarni hisoblashni bilamiz. Quyida biz bunduy ko'rinishdagi integrallarni hisoblashda noananaviy usuldan foydalanamiz.

2-misol. Integralni hisoblang. $\int\frac{3x+1}{2x^2-2x+3}dx$

Yechish .**1-usul**.Noananviy –innovatsion usul. Bizda

$$3x+1=A\frac{d}{dx}\left(2x^2-2x+3\right)+B$$

dan

$$3x+1=A(4x-2)+B=4Ax-2A+B$$

bo'lgani uchun

$$\begin{cases}4A=3,\\-4A+B=1\end{cases}\Rightarrow A=\frac{3}{4},B=\frac{5}{2}.$$

Bularga asosan

$$\int\frac{3x+1}{2x^2-2x+3}dx=\int\frac{\frac{3}{4}(4x-2)+\frac{5}{2}}{2x^2-2x+3}dx=\frac{3}{4}\int\frac{(4x-2)}{2x^2-2x+3}dx+\frac{5}{2}\int\frac{1}{2x^2-2x+3}dx=\frac{3}{4}I_1+\frac{5}{2}I_2.$$

Bu yerda

$$I_1=\int\frac{(4x-2)}{2x^2-2x+3}dx=\left|\begin{matrix}t=2x^2-2x+3\\dt=(4x-2)dx\end{matrix}\right|=\int\frac{dt}{t}=\ln|t|+C_1=\ln\left|2x^2-2x+3\right|+C_1.$$

$$\int\frac{1}{2(x^2-x+\frac{3}{2})}dx=\frac{1}{2}\int\frac{1}{(x^2-x+\frac{3}{2})}dx=\frac{1}{2}\int\frac{1}{\left(x-\frac{1}{2}\right)^2+\left(\frac{\sqrt{5}}{2}\right)^2}dx=\frac{1}{\sqrt{5}}arctg\frac{2x-1}{\sqrt{5}x}+C_2.$$

Demak

$$\int\frac{3x+1}{2x^2-2x+3}dx=\frac{3}{4}I_1+\frac{5}{2}I_2=\frac{3}{4}\ln\left|2x^2-2x+3\right|+\frac{1}{\sqrt{5}}arctg\frac{2x-1}{\sqrt{5}x}+C.$$

3-misol. Integralni hisoblang. $\int(3x-2)\sqrt{x^2+x+1}dx.$

Yechish .Noananaviy yangi o'zgaruvchi kiritish usuli.Quyidagi almashti-rishni bajaramiz

$$3x-2=A(2x+1)+B=2Ax+A+B$$

$$2A=3\Rightarrow A=\frac{3}{2,},A+B=-2\Rightarrow B=-\frac{7}{2}.$$

$$3x-2=\frac{3}{2}(2x+1)-\frac{7}{2}$$

$$I=\frac{3}{2}\int(3x-2)\sqrt{x^2+x+1}dx-\frac{7}{2}\int\sqrt{x^2+x+1}dx=\frac{3}{2}I_1-\frac{7}{2}I_2.$$

$$I_1=\int(3x-2)\sqrt{x^2+x+1}dx=\int\sqrt{x^2+x+1}(3x-2)dx=\left(x^2+x+1\right)^{3/2}+C_1.$$

$$\int\sqrt{x^2+x+1}dx=\int\sqrt{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{5}}{2}\right)^2}dx=\frac{2x+1}{4}\sqrt{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{5}}{2}\right)^2}+\ln\left|\frac{2x+1}{2}+\sqrt{x^2+x+1}\right|+C_2.$$

Demak

$$I=\frac{3}{2}I_1-\frac{7}{2}I_2=\frac{3}{2}\left(x^2+x+1\right)^{3/2}-\frac{7}{2}\frac{2x+1}{4}\sqrt{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{5}}{2}\right)^2}-\frac{3}{8}\ln\left|\frac{2x+1}{2}+\sqrt{x^2+x+1}\right|+C.$$

4-**misol**. Integralni hisoblang. $\int\sqrt{5-4x-x^2}\,dx$

Yechish .**1-usul**.Noananviy usul. Berilgan integralni quyidagicha yozamiz

$$\int\sqrt{5-\left(x^2+4x+4\right)+4}\,dx=\int\sqrt{9-(x+2)^2}\,dx=3\int\sqrt{1-\left(\frac{x+2}{3}\right)^2}\,dx$$

$$\frac{x+2}{3}=\sin\varphi,\ dx=3\cos\varphi d\varphi\,,$$

$$3\int\sqrt{1-\sin^2\varphi}\,3\cos\varphi d\varphi=9\int\left|\cos\varphi\right|\cos\varphi d\varphi=9\int\cos^2\varphi d\varphi=9\int\frac{1+\cos 2\varphi}{2}d\varphi=$$

$$=\frac{9}{2}\int(1+\cos 2\varphi)d\varphi=\frac{9}{2}\int(1+\cos 2\varphi)d\varphi=\frac{9}{2}\varphi+\frac{9}{4}\sin 2\varphi+C=\frac{9}{2}\arcsin\frac{x+2}{3}+\frac{9}{4}\sin 2\varphi+C=$$

$$\frac{9}{2}\arcsin\frac{x+2}{3}+\frac{9}{4}2\sin\varphi\cos\varphi+C=\frac{9}{2}\arcsin\frac{x+2}{3}+\frac{3x+6}{2}\sqrt{1-\left(\frac{x+2}{3}\right)^2}+C.$$

5-Misol.Integralni hisoblang. $\int\sqrt{\frac{x-1}{3-x}}\,dx.$

Yechish. **1-usul**. O'zgaruvchi almashtirish usuli.

$$\int\sqrt{\frac{x-1}{3-x}}\,dx=\left|\begin{array}{l}u=\sqrt{\frac{x-1}{3-x}},u^2=\frac{x-1}{3-x}\\ x=\frac{3u^2+1}{u^2+1}=1+\frac{2u^2+1}{u^2+1},dx=d\left(1+\frac{2u^2+1}{u^2+1}\right)\end{array}\right|=\frac{2u^3}{u^2+1}-2\int\frac{2u^3}{u^2+1}du=$$

$$=2u-\frac{2u}{u^2+1}-2\int 1du-\int\frac{1}{u^2+1}du=2u-\frac{2u}{u^2+1}-2u+2arctgu+C=$$

$$=-\frac{2u}{u^2+1}+2arctgu+C=\frac{2\sqrt{\frac{x-1}{3-x}}}{\frac{x-1}{3-x}+1}+2arctg\sqrt{\frac{x-1}{3-x}}+C=2arctg\sqrt{\frac{x-1}{3-x}}-\frac{\sqrt{(x-1)(3+x)}}{x-1+3-x}+C=$$

$$=2arctg\sqrt{\frac{x-1}{3-x}}-\sqrt{(x-1)(3+x)}+C.$$

2-usul.Sun'iy usul va trigonometrik almashtirish usuli.
bizda quyidagi uchburchakdan (2-chizma)

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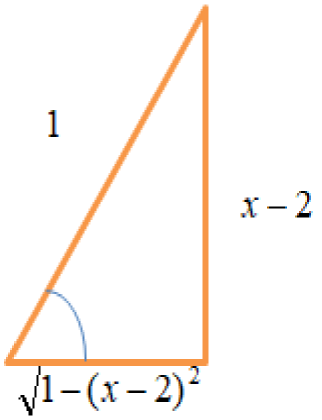
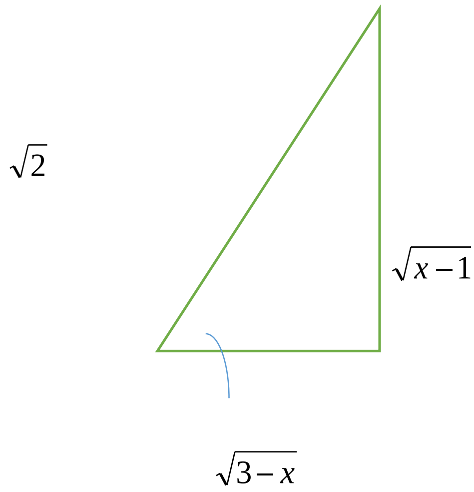


2-chizma



3-chizma

$$=\int \frac{x-1}{\sqrt{1-(x-2)^2}}dx=\int \frac{1+\sin t}{\sqrt{1-\sin^2 t}}\cos t dt=\int (1+\sin t)dt=t-\cos t+C=\arcsin(x-2)-\cos t+C=$$
$$\arcsin(x-2)-\sqrt{-x^2+4x-3}+C.$$

3-usul. Bizda 3- chizdadan $\sin^2\varphi-\cos^2\varphi=\frac{x-1-3+x}{2}=x-2$ bo’ladi.

$$\int\sqrt{\frac{x-1}{3-x}}dx=\left|\begin{array}{l} tg\varphi=\sqrt{\frac{x-1}{3-x}} \\ \sin\varphi=\sqrt{\frac{x-1}{2}},\cos\varphi=\sqrt{\frac{3-x}{2}} \end{array}\right|=\int tg\varphi d(\sin^2\varphi-\cos^2\varphi)=$$

$$\int tg\varphi d(2\sin\varphi\cos\varphi+2\cos\varphi\sin\varphi)=4\int tg\varphi\sin\varphi\cos\varphi=4\sin^2\varphi d\varphi=2\int(1-\cos 2\varphi)d\varphi=$$

$$=2\varphi-2\sin\varphi\cos\varphi=2arctg\sqrt{\frac{x-1}{3-x}}-2\sqrt{\frac{x-1}{3-x}}\frac{\sqrt{3-x}}{2}+C=2arctg\sqrt{\frac{x-1}{3-x}}-\sqrt{(x-1)(3-x)}+C.$$

6-misol. Integralni hisoblang. $\int\frac{x+\sin x}{1+\cos x}dx.$

Yechish. **1-usul**.Agar biz $\sin x=2\sin\frac{x}{2}\cos\frac{x}{2}$ va $1+\cos x=2\cos^2\frac{x}{2}$ ayniyatlarni e’tiborga olsak quyidagilarga ega bo’lamiz

$$I=\int\frac{x+\sin x}{1+\cos x}dx=\int\frac{x+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}dx=\int\frac{\frac{x}{2}+\sin\frac{x}{2}\cos\frac{x}{2}}{\cos^2\frac{x}{2}}dx=\int\frac{\sin^2\frac{x}{2}\left(\frac{x}{2}\cos ec^2\frac{x}{2}+ctg\frac{x}{2}\right)}{\sin^2\frac{x}{2}ctg^2\frac{x}{2}}dx=$$

$$=\int \frac{\left(\frac{x}{2}\operatorname{cos}ec^{2}\frac{x}{2}+ctg\frac{x}{2}\right)}{ctg^{2}\frac{x}{2}}dx=\int \frac{ctg\frac{x}{2}.1-x\left(-\frac{1}{2}\operatorname{cos}ec^{2}\frac{x}{2}\right)}{ctg^{2}\frac{x}{2}}dx=\int \frac{ctg\frac{x}{2}\frac{d}{dx}(x)-x\frac{d}{dx}\left(ctg\frac{x}{2}\right)}{ctg^{2}\frac{x}{2}}=$$

$$\int \frac{d}{dx}\left(\frac{x}{ctg\frac{x}{2}}\right)=\frac{x}{ctg\frac{x}{2}}+C=xtg\frac{x}{2}+C.$$

2-**usul.**Quyidagicha yozamiz

$$\int \frac{x+\sin x}{1+\cos x}dx=\int \frac{x}{1+\cos x}dx+\int \frac{\sin x}{1+\cos x}dx=I_1+I_{2.}$$

$$I_1=\int \frac{\sin x}{1+\cos x}dx=\int \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}dx=\int \frac{\sin\frac{x}{2}\cos\frac{x}{2}}{\cos^2\frac{x}{2}}dx=\int tg\frac{x}{2}dx=\left|\begin{matrix}u=\frac{x}{2}\\ du=\frac{dx}{2}\end{matrix}\right|=2\int tgudu=$$

$$=-2\ln|\cos u|+C=-2\ln\left|\cos\frac{x}{2}\right|+C.$$

$$I_2=\int \frac{x}{1+\cos x}dx=\frac{1}{2}\int x\sec^2\frac{x}{2}dx=\left|\begin{matrix}u=\frac{x}{2}\\ dx=2du\end{matrix}\right|=\frac{1}{2}\int 2u\sec^2 udu$$

$A=u, dB=\sec^2 u$ bo’lsin , integralni bo’laklab integrallaymiz

$dA=du, B=tgu$ deb belgilaymiz va bundan esa

$$\int u\sec^2 udu=utgu-\int utgudu.=utgu+\ln|\cos u|$$

yoki

$$I_2=\int \frac{x}{1+\cos x}dx=xtg\frac{x}{2}+2\ln\left|\frac{x}{2}\right|$$

natijani olamiz. Natijada

$$\int \frac{x+\sin x}{1+\cos x}dx=-2\ln\left|\cos\frac{x}{2}\right|+xtg\frac{x}{2}+2\ln\left|\frac{x}{2}\right|+C$$

ni olamiz.

4-misol. Integralni hisoblang. $\int\limits_{-1}^{3}\sqrt{3+2x-x^2}\,dx.$

Yechish. 1-usul. Trigonometrik almashtirish usuli.

$$\int\limits_{-1}^{3}\sqrt{3+2x-x^2}\,dx=\int\limits_{-1}^{3}\sqrt{4-(x-2)^2}\,dx=\left|\begin{matrix}2\sin\alpha=x-2\\ \sqrt{4-(2\sin\alpha)^2}=2\cos\alpha=x-1\end{matrix}\right|$$

Bu yerda, agar $x=-1$ bo'lsa $\alpha=-\frac{\pi}{2}$, agar $x=3$ bo'lsa $\alpha=\frac{\pi}{2}$ ga teng bo'ladi. Demak

$$\int_{-1}^{3}\sqrt{3+2x-x^2}\,dx=\int_{-1}^{3}\sqrt{4-(x-2)^2}\,dx=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sqrt{4-(2\sin\alpha)^2}\,d\alpha=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}2\sqrt{1-\sin^2\alpha}\,2\cos\alpha d\alpha=$$

$$4\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos^2\alpha d\alpha=4\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{1+\cos 2\alpha}{2}\,d\alpha=8(\alpha+\frac{1}{2}\sin 2\alpha)\Big|_0^{\frac{\pi}{2}}=2\pi.$$

2-usul. Geometrik usul. Integral ostidagi funksiya $y=\sqrt{3+2x-x^2}$ ni $(x-1)^2+y^2=4$ ko'rinishda yozamiz .Bu esa markazi $C(1;0)$ nuqtada va radiusi $R=2$ ga teng bo'lgan aylananing tenglamasidan iborat.Bizda berilgan aniq integralning qiymati ushbu aylananing yuqori yarim qismi bilan chegaralangan yuzani beradi.

Demak izlanayotgan yuza $S=\frac{1}{2}\pi R^2=2\pi$ ga teng ekan.

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