STABILITY OF AN ELASTIC ROD WITH DYNAMIC ABSORBERS UNDER HARMONIC TRANSVERSE VIBRATIONS

Anvar Kudratov, Shokhzod Khamitov
Uzbek-Finish pedagogical institute

Abstract. This paper considers the stability problem of a simply supported rod with two dynamic vibration absorbers and hysteretic energy dissipation. Linearization of dependencies describing hysteretic energy dissipation due to imperfect elasticity of the material is carried out using Pisarenko-Boginich functional in a frequency-independent form

Keywords: stability, rod, oscillations, kinematic excitations, linearization

INTRODUCTION

When studying the oscillatory movements of the mechanical systems under consideration, it is often necessary to pay attention to the parameters of the system, which make it possible, to one degree or another, to predict the desired movements.

Thus, there are many works [1-5] on the study of vibrations of mechanical systems with distributed parameters under the influence of excitations and forces in different directions, with different parameters of the mechanical system, as well as damping harmful vibrations [6-8], methods for modeling mechanical systems with dynamic absorber.

It is known that nonlinear oscillations of mechanical systems have special mechanical effects. Thus, in [9-12], vibrations of elastic mechanical systems with nonlinear elastically dissipative characteristics are considered, where it is shown that such mechanical systems have highly absorbing properties reflected by a hysteresis loop. But on the other hand, unstable oscillations are possible in such systems in the form of an amplitude jump with the slightest changes in the oscillation frequency, which proceeds from the fact that the frequencies of nonlinear oscillations depend on the amplitudes. Methods of equivalent and harmonic linearization in frequency-dependent and frequency-independent forms are presented. The main types of the function characterizing the nonlinear dependence of stresses on deformations due to imperfect elasticity of the material in the structural elements of the system, as well as nonlinear dependencies between tensions and deformations, are given. The nonlinear dependencies are based on the models proposed by N.N.Davydenkov, G.S.Pisarenko, and O.E.Boginich. The stability of some oscillatory mechanical systems is considered.

In [13-15], the problems of stability of stationary vibrations of mechanical systems with elastic-dissipative characteristics of the hysteresis type under kinematic and random excitations are considered. The conditions and areas of stability of the considered systems are determined.

METHOD OF RESEARCH

In this paper, the stability of transverse vibrations of an elastic rod with two

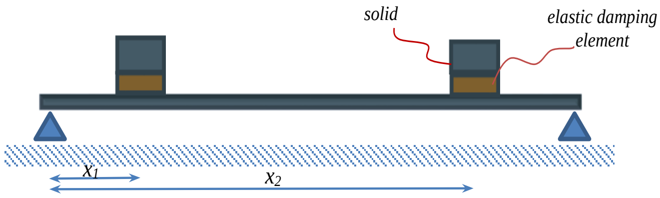

FIGURE 1. Rod with two dynamic absorbers

dynamic vibration absorbers with elastic-dissipative characteristics of the hysteresis type is researched. Based on the system of differential equations of motion, using the method of slowly varying amplitudes, the system is brought to a normal form and the amplitude-frequency characteristics of the system under consideration are found. The characteristic equations are obtained, the stability conditions according to the Hurwitz criterion are verified. In particular, graphs of expressions in these inequalities are constructed. Stability conditions are analytically expressed depending on the system parameters. The change in stability conditions is analyzed depending on changes in the masses of dynamic absorbers, changes in the stiffness of the elastic elements of dynamic absorbers and changes in the installation locations of dynamic absorbers.

The differential equations of a rod and two dynamic absorbers with hysteresis energy dissipation under kinematic excitation are written as follows:

$$\frac{\partial^2 M}{\partial x^2} + \rho F \frac{\partial^2 w}{\partial t^2} - c_1 R_1 \delta_1 (x - x_1)\zeta_1 - c_2 R_2 \delta_2 (x - x_2)\zeta_2 = -\rho F \frac{\partial^2 w_0}{\partial t^2};$$
$$m_1 \frac{\partial^2 w(x_1)}{\partial t^2} + m_1 \frac{\partial^2 \zeta_1}{\partial t^2} + c_1 R_1 \zeta_1 = -m_1 \frac{\partial^2 w_0}{\partial t^2}; \tag{1}$$
$$m_2 \frac{\partial^2 w(x_2)}{\partial t^2} + m_2 \frac{\partial^2 \zeta_2}{\partial t^2} + c_2 R_2 \zeta_2 = -m_2 \frac{\partial^2 w_0}{\partial t^2},$$

where M – bending moment; ρ, F – the density of the material and the cross-sectional area of the rod, respectively; w – rod deflection function; w_0 –displacement of the base; x_1, x_2; $w(x_1), w(x_2)$ – coordinates and displacements at the points of the rod in which the dynamic absorbers are installed; c_1, c_2 – stiffness coefficients of the elastic damping elements of the dynamic absorbers; m_1, m_2 – dynamic absorbers masses; ζ_1, ζ_2 – displacements of the dynamic absorbers relative to the rod; $\delta_1(x - x_1), \delta_2(x - x_2)$ – Dirac delta functions;

$$R_1 = 1 + (-\nu_1 + j\nu_2)[D_0 + f(\zeta_{1ot})]; \quad R_2 = 1 + (-\eta_1 + j\eta_2)[E_0 + g(\zeta_{2ot})]. \tag{2}$$

Here $j^2 = -1$; $\nu_1, \nu_2, \eta_1, \eta_2$ – coefficients depending on the dissipative properties of the materials of the elastic damping elements of the dynamic absorbers, determined from the corresponding dependencies of the contour of the hysteresis loop; $f(\zeta_{1ot}), g(\zeta_{2ot})$ - the decrements of oscillations, represented in general terms, as functions of the maximum (amplitude) values of relative deformation ζ_{1ot}, ζ_{2ot} :

$$f(\zeta_{1ot}) = \sum D_{K_1}\zeta_{1ot}^{K_1}; \quad (3)$$

$$g(\zeta_{2ot}) = \sum E_{K_2}\zeta_{2ot}^{K_2}, \quad (4)$$

$D_0, D_1, \ldots, D_{r_1}, E_0, E_2, \ldots, E_{r_2}$ – some numbers (parameters) of the hysteresis loop depending on the damping properties of the materials of the elastic damping elements of the dynamic absorbers and determined by the experimental selected curves $\delta_1 = f(\zeta_{1ot})$ and $\delta_2 = g(\zeta_{2ot})$ points with coordinates $\delta_{1i}, (\zeta_{1ot})_i$ and $\delta_{2i}, (\zeta_{2ot})_i$ accordingly.

$$EJ\frac{\partial^4 w}{\partial x^4} + \rho F\frac{\partial^2 w}{\partial t^2} - c_1R_1\delta_1(x-x_1)\zeta_1 - c_2R_2\delta_2(x-x_2)\zeta_2 = -\rho F\frac{\partial^2 w_0}{\partial t^2};$$

$$m_1\frac{\partial^2 w(x_1)}{\partial t^2} + m_1\frac{\partial^2\zeta_1}{\partial t^2} + c_1R_1\zeta_1 = -m_1\frac{\partial^2 w_0}{\partial t^2}; \quad (5)$$

$$m_2\frac{\partial^2 w(x_2)}{\partial t^2} + m_2\frac{\partial^2\zeta_2}{\partial t^2} + c_2R_2\zeta_2 = -m_2\frac{\partial^2 w_0}{\partial t^2}.$$

Where J - moment of inertia of the cross section of the rod.

To solve this system, the method of Fourier was used

$$w(x,t) = \sum_{i=1}^{\infty} u_i(x)q_i(t).$$

After some calculations, the system (1) is reduced to the form

$$\ddot{q}_i + p_i^2 q_i - \mu_1\mu_{0i}n_1^2 u_{i1}R_1\zeta_1 - \mu_2\mu_{0i}n_2^2 u_{i2}R_2\zeta_2 = -d_iW_0;$$

$$u_{i1}\ddot{q}_i + \ddot{\zeta}_1 + n_1^2R_1\zeta_1 = -W_0; \quad (6)$$

$$u_{i2}\ddot{q}_i + \ddot{\zeta}_2 + n_2^2R_2\zeta_2 = -W_0.$$

where p_i – the own frequency of the rod; $\mu_1 = \frac{m_1}{m_c}$; $\mu_2 = \frac{m_2}{m_c}$; $\mu_{0i} = \frac{1}{d_{2i}}$; $d_i = \frac{d_{1i}}{d_{2i}}$; $d_{1i} = \int_0^l u_i dx$; $d_{2i} = \int_0^l u_i^2 dx$; $m_c = \rho Fl$ – rod mass; m_1, m_2 – masses of dynamic absorbers; $u_i(x)$ – the rod vibration's own shapes; W_0 – acceleration of the base, $u_{i1} = u_i(x_1)$; $u_{i2} = u_i(x_2)$; $n_1 = \sqrt{\frac{c_1}{m_1}}$, $n_2 = \sqrt{\frac{c_2}{m_2}}$; c_1, c_2; ζ_1, ζ_2 – oscillation frequencies; stiffness coefficients of elastic elements and relative displacements of dynamic absorbers.

Acceleration of the base during harmonic oscillations is

$$W_0 = w_0\cos\omega t,$$

where w_0 – the amplitude value of the acceleration; ω – frequency.

RESEARCH RESULTS

The transfer functions of system (2) are found in the form

$$q(j\omega) = -\frac{B_1(\omega) + jB_2(\omega)}{B_3(\omega) + jB_4(\omega)} \cdot w_0;$$

$$\zeta_1(j\omega) = -\frac{B_5(\omega) + jB_6(\omega)}{B_3(\omega) + jB_4(\omega)} \cdot w_0;$$
$$\zeta_2(j\omega) = -\frac{B_7(\omega) + jB_8(\omega)}{B_3(\omega) + jB_4(\omega)} \cdot w_0;$$

where

$$B_1(\omega) = d_i\omega^4 - [T_1 n_1^2(1 - \nu_1 N_1) + T_2 n_2^2(1 - \theta_1 N_2)]\omega^2 +$$
$$+ n_1^2 n_2^2 T_3[(1 - \nu_1 N_1)(1 - \theta_1 N_2) - \nu_2\theta_2 N_1 N_2];$$
$$B_2(\omega) = -[T_1 n_1^2 \nu_2 N_1 + T_2 n_2^2 \theta_2 N_2]\omega^2 + n_1^2 n_2^2 T_3[(1 - \nu_1 N_1)\theta_2 N_2 +$$
$$+ (1 - \theta_1 N_2)\nu_2 N_1];$$
$$B_3(\omega) = -\omega^6 + [p_i^2 + n_1^2 T_6(1 - \nu_1 N_1) + n_2^2 T_7(1 - \theta_1 N_2)]\omega^4 -$$
$$-\{n_1^2 p_i^2(1 - \nu_1 N_1 - \eta_c \nu_2 N_1) + n_2^2 p_i^2(1 - \theta_1 N_2) +$$
$$+ n_1^2 n_2^2 T_8[(1 - \nu_1 N_1)(1 - \theta_1 N_2) - \nu_2\theta_2 N_1 N_2]\}\omega^2 +$$
$$+ n_1^2 n_2^2[(1 - \nu_1 N_1)(1 - \theta_1 N_2) - \nu_2\theta_2 N_1 N_2];$$
$$B_4(\omega) = [n_1^2 T_6 \nu_2 N_1 + n_2^2 T_7 \theta_2 N_2]\omega^4 - \{n_1^2 p_i^2[\eta_c(1 - \nu_1 N_1) + \nu_2 N_1] +$$
$$+ n_2^2 p_i^2 \theta_2 N_2 + n_1^2 n_2^2 T_8[(1 - \nu_1 N_1)\theta_2 N_2 +$$
$$+ (1 - \theta_1 N_2)\nu_2 N_1]\}\omega^2 + n_1^2 n_2^2[(1 - \nu_1 N_1)\theta_2 N_2 + (1 - \theta_1 N_2)\nu_2 N_1];$$
$$B_5(\omega) = (1 - d_i u_{i1})\omega^4 - [p_i^2 + T_4 n_2^2(1 - \theta_1 N_2)]\omega^2 + n_2^2 p_i^2[1 - \theta_1 N_2];$$
$$B_6(\omega) = -n_2^2 T_4 \theta_2 N_2 \omega^2 + n_2^2 p_i^2 \theta_2 N_2;$$
$$B_7(\omega) = (1 - d_i u_{i2})\omega^4 - [p_i^2 + T_5 n_1^2(1 - \nu_1 N_1)]\omega^2 + n_1^2 p_i^2[1 - \nu_1 N_1];$$
$$B_8(\omega) = -(\eta_c p_i^2 + n_1^2 T_5 \nu_2 N_1)\omega^2 + n_1^2 p_i^2 \nu_2 N_1;$$

where

$$T_1 = d_i + \mu_{0i}\mu_1 u_{i1};\; T_2 = d_i + \mu_{0i}\mu_2 u_{i2};\; T_3 = d_i + \mu_{0i}(\mu_1 u_{i1} + \mu_2 u_{i2});$$
$$T_4 = 1 + \mu_{0i}\mu_2 u_{i2}(u_{i2} - u_{i1}) - u_{i1} d_i;\; T_5 = 1 + \mu_{0i}\mu_1 u_{i1}(u_{i1} - u_{i2}) - u_{i2} d_i;$$
$$T_6 = 1 + \mu_{0i}\mu_1 u_{i1}^2;\; T_7 = 1 + \mu_{0i}\mu_2 u_{i2}^2;\; T_8 = 1 + \mu_{0i}(\mu_1 u_{i1}^2 + \mu_2 u_{i2}^2).$$

The amplitude-frequency characteristics are obtained by calculating the absolute values of the transfer functions.

Solutions of the system (2) are sought in the form of

$$q_i = a_i \cos(\omega t + \alpha_i);$$
$$\zeta_1 = b_1 \cos(\omega t + \beta_1); \qquad (7)$$
$$\zeta_2 = b_2 \cos(\omega t + \beta_2),$$

where the amplitudes and phases $a_i, \alpha_i, b_1, \beta_1, b_2, \beta_2$ of the oscillations are considered as slowly varying.

Substituting (7) into equations (6), equating the second derivatives to zero, we find the equations of motion in normal form

$$\dot{a}_i = -\frac{1}{2\omega}[-d_i w_0 \sin\alpha_i - \mu_1\mu_{0i} n_1^2 u_{i1} b_1 H_1 - \mu_2\mu_{0i} n_2^2 u_{i2} b_2 \sin\varphi_2];$$
$$\dot{\alpha}_i = -\frac{1}{2\omega a_i}\big[-d_i w_0 \cos\alpha_i + \left(\omega^2 - p_i^2\right)a_i + \mu_1\mu_{0i} n_1^2 u_{i1} b_1 H_2$$
$$+ \mu_2\mu_{0i} n_2^2 u_{i2} b_2 \cos\varphi_2\big];$$

$$\dot{b_1} = -\frac{1}{2\omega}\left[-(1-d_i u_{i1})w_0 \sin\beta_1 + n_1^2(1+\mu_1\mu_{0i}u_{i1}^2)\nu_2 N_1 b_1 + u_{i1}a_i p_i^2 \sin\varphi_1 + \mu_2\mu_{0i}n_2^2 u_{i1}u_{i2}b_2 \sin\varphi_3\right];$$

(8)

$$\dot{\beta_1} = -\frac{1}{2\omega b_1}\left[-(1-d_i u_{i1})w_0 \cos\beta_1 + \left(\omega^2 - n_1^2(1+\mu_1\mu_{0i}u_{i1}^2)(1-\nu_1 N_1)\right)b_1 + u_{i1}a_i p_i^2 \cos\varphi_1 - \mu_2\mu_{0i}n_2^2 u_{i1}u_{i2}b_2 \cos\varphi_3\right];$$

$$\dot{b_2} = -\frac{1}{2\omega}\left[-(1-d_i u_{i2})w_0 \sin\beta_2 + u_{i2}a_i p_i^2 \sin\varphi_2 - \mu_1\mu_{0i}n_1^2 u_{i1}u_{i2}b_1 \sin\varphi_3\right];$$

$$\dot{\beta_2} = -\frac{1}{2\omega b_2}\left[-(1-d_i u_{i2})w_0 \cos\beta_2 + \left(\omega^2 - n_2^2(1+\mu_2\mu_{0i}u_{i2}^2)\right)b_2 + u_{i1}a_i p_i^2 \cos\varphi_2 - \mu_1\mu_{0i}n_1^2 u_{i1}u_{i2}b_1 \cos\varphi_3\right],$$

where

$$H_1 = (1-\nu_1 N_1)\sin\varphi_1 + \nu_2 N_1 \cos\varphi_1;\ H_2 = (1-\nu_1 N_1)\cos\varphi_1 - \nu_2 N_1 \sin\varphi_1;$$

$$\varphi_1 = \beta_1 - \alpha_i, \varphi_2 = \beta_2 - \alpha_i, \varphi_3 = \beta_2 - \beta_1.$$

To study the stability of stationary oscillations of the system, we will use the Lyapunov method as a first approximation. By varying equations (8), it is possible to obtain a system of equations in variations, from which we obtain the characteristic equation

$$\lambda^6 + A_1\lambda^5 + A_2\lambda^4 + A_3\lambda^3 + A_4\lambda^2 + A_5\lambda + A_6 = 0, \quad (9)$$

in this case, the Hurwitz criterion will look like

$$A_1 > 0, \quad A_1A_2 - A_3 > 0, \quad -A_1^2A_4 + A_1A_2A_3 + A_1A_5 - A_3^2 > 0,$$

$$A_1^2A_2A_6 - A_1^2A_4^2 - A_1A_2^2A_5 + A_1A_2A_3A_4 - A_1A_3A_6 + 2A_1A_4A_5 + A_2A_3A_5 - A_3^2A_4 - A_5^2 > 0,$$

$$-A_1^3A_6^2 + 2A_1^2A_2A_5A_6 + A_1^2A_3A_4A_6 - A_1^2A_4^2A_5 - A_1A_2^2A_5^2 - A_1A_2A_3^2A_6 + A_1A_2A_3A_4A_5 - 3A_1A_3A_5A_6 + 2A_1A_4A_5^2 - A_2A_3A_5^2 + A_3^3A_6 - A_3^2A_4A_5 - A_5^3 > 0, \quad (10)$$

$$(-A_1^3A_6^2 + 2A_1^2A_2A_5A_6 + A_1^2A_3A_4A_6 - A_1^2A_4^2A_5 - A_1A_2^2A_5^2 - A_1A_2A_3^2A_6 + A_1A_2A_3A_4A_5 - 3A_1A_3A_5A_6 + 2A_1A_4A_5^2 + A_2A_3A_5^2 + A_3^3A_6 - A_3^2A_4A_5 - A_5^3)A_6 > 0.$$

For numerical analysis, let's take a rod with dimensions $l = 25\cdot10^{-2}$ m; $b = 10^{-2}$ m; $h = 2\cdot10^{-3}$ m, made of steel grade 40X, masses and installation coordinates of the dynamic absorbers are $m_1 = m_2 = 0.0027335$ kg; $x_1 = \frac{l}{3}$, $x_2 = \frac{2l}{3}$. Coefficients $\nu_1 = \frac{3}{8}$; $\nu_2 = \frac{1}{2\pi}$; $\nu_2 N_1 = 817684.6\cdot a_{1*}^2 - 1.920207\cdot10^{12}\cdot a_{1*}^4 + 1.556859\cdot10^{18}\cdot a_{1*}^6$.

The first four inequalities of the six inequalities of the Hurwitz criterion are satisfied. The graphs of the coefficients of the last two inequalities depending on the change in the stiffness of the elastic elements of the dynamic absorbers are shown in Figure 2. It can be seen from the graphs that the functions of these inequalities have negative values, which indicates the presence of unstable stationary amplitudes.

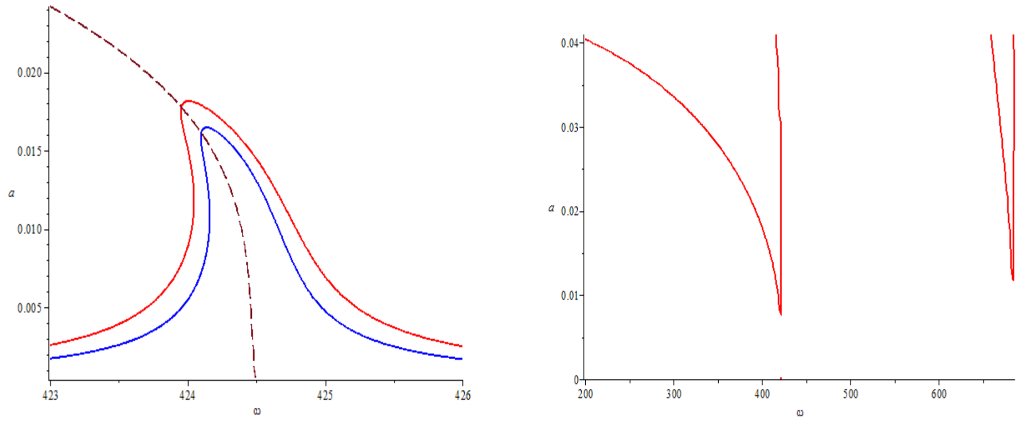

FIGURE 2. Graphs of the last two terms of Hurwitz inequalities

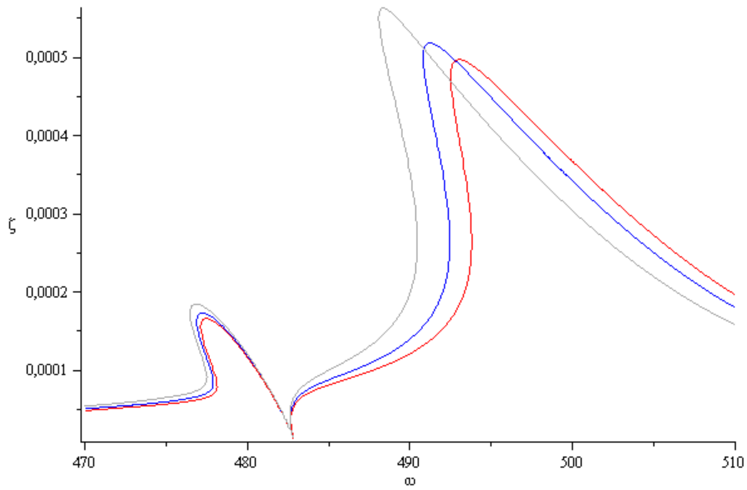

FIGURE 3. Amplitude-Frequency Characteristics of the vibrations of a rod with two dynamic absorbers is the ratio of the masses of the dynamic absorbers and the rod, respectively, 0.03 (gray), 0.033 (blue) and 0.035 (red)

In Figure 3, graphs of amplitude-frequency characteristics are plotted depending on the ratio of the masses of the dynamic absorbers and the rod for the first oscillation form. From Fig.3 it can be concluded that with an increase in the ratio of the masses of the dynamic absorbers and the rod from 0.03 to 0.035, the maximum values of the oscillation amplitudes decrease. At the same time, areas of unstable amplitudes remain. The frequencies of unstable amplitudes depart from the antiresonance frequency.

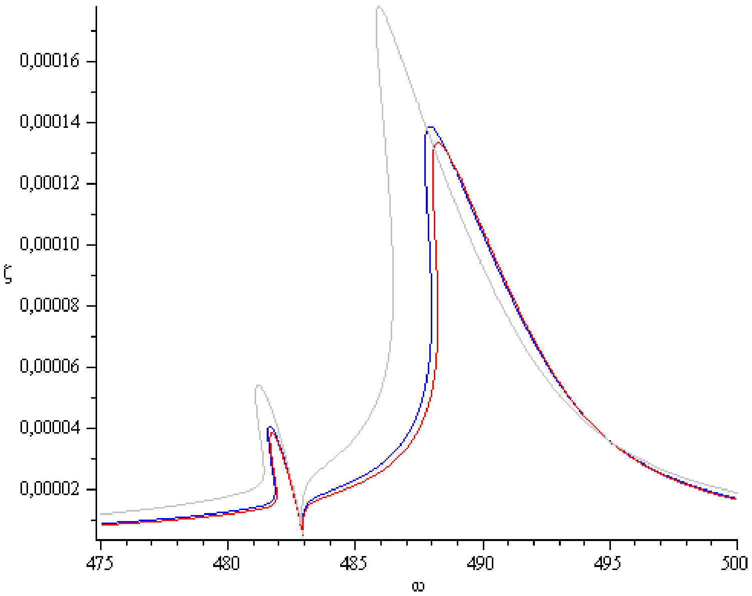

FIGURE 4. Amplitude-Frequency Characteristics of vibrations of a rod with two dynamic absorbers with a change in installation locations: red curve for $x_{1,2}$=0,5l±0,11l, blue curve for $x_{1,2}$=0,5l±0,09l and gray curve two dynamic absorbers combined into one

The graphs of the Amplitude-Frequency Characteristics of the considered system with different installation locations of dynamic absorbers are constructed. Thus, in Fig. 4, the Amplitude-Frequency Characteristics of a rod with two dynamic absorbers is constructed to dampen the first form of vibration near the center of the rod and in the center of the rod. It can be seen from the figure that if two dynamic dampers are combined into one, the oscillation amplitudes will increase much more than in the case when the dynamic dampers are located separately.

CONCLUSIONS

Differential equations of a kinematically excited rod with two dynamic vibration dampers and hysteresis energy dissipation in normal form are constructed. The amplitude-frequency characteristics of this system are found. It is shown that some terms of the Hurwitz inequalities have negative values for the stability of the system under consideration, which indicate the presence of unstable stationary amplitudes under certain conditions in the form of an abrupt change in oscillation amplitudes. These results ensure accuracy in the selection of these rod parameters in practical projects, when checking the dynamics and stability of the system in mathematical modeling.

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